Black Sea Grain Initiative: a Game-Theoretic Analysis

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October 13, 2023

Abstract

In July 2023, the Black Sea Grain Deal expired just a year after its inception. It has become a very popular topic due to its significant implications for grain and staple food prices. There has been an abundance of empirical analysis to understand the situation, but the recent developments in the Black Sea Grain Deal have not been examined using a game-theoretic approach. This paper provides a game-theoretic viewpoint of the Black Sea Grain Deal with the focus on the breakdown. Using principles of game theory, I develop an infinitely repeated game with a defined set of players, actions, and preferences expressed through payoffs. By analyzing the game for sub-game perfect Nash equilibria, there is a clearer understanding of the breakdown of the Black Sea Grain Deal and its future implications. I finish by discussing possible extensions and variations of the model along with what conditions need to be met for a game-theoretic approach to be viable in general international relations settings.

1 Introduction

On February 2022, Russia began its full-scale invasion of Ukraine. Ukraine's ability to export has been severely hampered by Russia's invasion [UNC22]. Before the war, 90% of Ukrainian crop exports went through ports at the Azov and Black Seas which became inaccessible due to Russian aggression [OEC22]. On July 2022, Turkey, Russia, Ukraine, and the U.N. signed the Black Sea Grain Deal. The deal allowed Ukraine to safely export grain, other food, and fertilizer from three Black Sea ports: Chornomorsk, Odesa, and Yuzhny/Pivdennyi [UNC22]. Along with the Black Sea Grain Initiative, the U.N. established an agreement with Russia "to facilitate the unimpeded exports to world markets of Russian food and fertilizer (including the raw materials required to produce fertilizers) to world markets" [Ped23]. The UN brokered these two deals with the aim of lowering food prices. To some extent, the deal was successful in its

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goal to reduce food prices [UNC22]. However, the deal had a finite term limit of 120 days.

The deal first naturally expired on November 2022. After consideration and further discussion with the UN, Russia agreed to continue the Black Sea Grain Initiative for another 120 days. The FAO Price Index continued to decrease. On March 2023, the U.N. met with Russia to discuss another extension. Moscow's agreement was contingent on the removal of Western sanctions. This led to a stall in the deal on May 2023, which required further talks that led to another 60-day extension [Ped23]. On July 2023, Russia said they were suspending co-operation with the deal once it reached its expiration date. Russia said that the agreement concerning their food and fertilizer exports must be met first before returning to the deal. For that, they have demanded that the Russian Agricultural Bank is reconnected to the SWIFT payment system and that restrictions hampering their agricultural exports (i.e. shipping, insurance) are lifted [Nic23] [Bon23]

This breakdown has been primarily studied from an empirical standpoint, with a particular focus on sanctions and restrictions, to explain why the Black Sea Grain Deal broke down [HS23] [Bon23]. However, there are still lessons to be learned from a game theoretic approach. By doing so, I conduct a game-theoretic analysis, where game theory is defined in [Rub94] as "a bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact. The basic assumptions that underlie the theory are that decision-makers pursue well-defined exogenous objectives (they are rational) and take into account their knowledge or expectations of other decision-makers behavior (they reason strategically)" (p.1). In response to complex, real-world phenomena, game theory provides a simple and clean structure that can be used to analyze well defined equilibrium outcomes. Game theory helps understand outcomes concerning decision-makers whose outcomes are interdependent on others' actions. Therefore, game theory is a powerful tool of analyzing international relations. The Black Sea Grain Initiative has clear actors who have certain actions and preferences that inform those very actions. Structuring a game around that can give clear, decisive outcomes concerning the agreement's breakdown. The merits of game-theoretic analysis will be discussed in more detail in the literature review.

2 Literature Review

The breakdown of the Black Sea Grain Deal is clearly an issue of international relations. The literature has thoroughly explored the links between game theory and international relations in general. [Cor01] explores the variety of international relations scenarios in which a game-theoretic approach could be utilized. With a focus on interaction between nation-states, the primary issues are security and economics. Like most economic fields, game theory is founded in the principal influence of individual rationality, meaning that players or actors always play with the aspiration to maximize their own individual payoffs. Applying this to an international relations context, countries will take the action that most benefits themselves.

The area of [Cor01] that is of primary interest is the discussion of international crises (p.193-195). International crises are characterized specifically "by the events that take place when one or more nation-states perceive that their security is suddenly, immediately and seriously threatened by actions proposed or performed by other nation-states or by events accidentally taking place in them" (p.193). [Cor01] boils these crises down to two actions: confrontation and cooperation where the "threatening nation-state" attempts to force the "threatened nation-state" to follow their demands. Simultaneously, the "threatened nation-state" is trying to make the other nation stop their demands. Other papers explore specific crises in detail.

[Zag14] analyzes several games with varying structures, actions and payoffs that attempt to model the Cuban Missile Crisis (CMC). Of the models which [Zag14] has examined, the one most similar to this paper's aims is Thomas Schelling's 1966 Chicken Game (p.22-26) where the worst outcome is mutual defection and so one player would yield to the other. Schelling believed whatever side of the CMC pushed the issue first would force the other to capitulate and "swerve," gaining the advantage. This led Schelling to attribute the U.S.'s 'victory' to Kennedy threatening brinkmanship. [Zag14] explains how Schelling's model was later proven wrong using White House tapes. The tapes showed Kennedy wanted to use blockades as a way to buy time for renegotiation (p.24). [Zag14] shows the significance of the real-world context of the crises being modelled. New discoveries and developments of understanding of a crisis can debunk a model that was previously supported. [Zag14] demonstrates how models have been developed to explain real-world events, not limited to but including the Cuban Missile Crisis. This is very applicable in modelling and understanding the Black Sea Grain Deal's breakdown.

A key component of the situation around the Black Sea Grain Deal was the several expirations and subsequent renegotiations that occurred. Frequent renegotiation in international agreements makes models of repeated games a suitable tool of analysis for agreements and their breakdowns. [Pea91], [Sla04], and [GT20] thoroughly explore the technical aspects of repeated games with discounted payoffs. [GT20] covers several variations of repeated games concerning monitoring and information while [Pea91] primarily focuses on repeated games concerning self-enforced agreements referencing several proofs and folk theorems concerning repeated games, sufficient patience/discount factors and defining repeated games and their equilibria. [GT20] and [Pea91] provide a vital, mathematical understanding of repeated games and their equilibria. [Sla04] tackles specific strategies used in repeated games such as grim trigger, tit for tat, limited retaliation, deviate once (DEV1L), Grim DEV1L, and Pavlov. He takes it a step further and assess whether combinations of the some of the aforementioned strategies could be supported as sub game perfect equilibria (2004).

[Kan08] specifically focuses on how repeated games are a setting that encourages mutual cooperation. [Kan08] highlights a key issue in international agreements; oftentimes, there is no body powerful enough to enforce an international agreement. With there being no explicit commitment device within the terms of the Black Sea Grain Deal, there is no external force mandating both sides to cooperate. So, it is best to model the deal in terms of a non-cooperative game. [Kan08] adds that a long-term relationship with several interactions is an environment most suitable to establish mutual cooperation, especially when formal contracts are too costly or impossible to enforce.

3 Model

3.1 Model Setup

Although the Black Sea Grain Initiative was signed by the U.N., Russia, Ukraine and Turkey, the model incorporates two players: Russia and the U.N. Turkey has similar interests to the U.N. which makes their payoffs identical. The U.N having more power and influence and Turkey makes the U.N the primary player between the two. As a result, their combined preferences are modeled as a single actor's, which is simply called the U.N. Although the deal concerns Ukrainian exports, Ukraine is essentially a bystander in the Black Sea Grain Deal. Unlike the U.N., they do not have the power to control the restrictions on Russia, meaning they do not have any action spaces in this game. These considerations also allow the use of more standardized games that would not be viable with more than two players. Although in real terms, negotiations of international agreements can be very complex, for the sake of modelling, I believe it is appropriate to collapse each actor's action space into two actions. They both have essentially binary choices. For the U.N, they can either offer concessions to Russia or not. And for Russia, they can either return to the Black Sea Grain Deal or not.

For the U.N., cooperating entails renegotiating the deal and giving some concessions. Defecting would mean the U.N. ends negotiations over the Black Sea Grain Initiative and provides no concessions to Russia. For Russia, cooperating entails returning to the Black Sea Grain Initiative. Defecting would mean Russia does not return to the Grain Deal.

The mutually beneficial outcome would be both sides cooperating and renewing a renegotiated Black Sea Grain Deal. Both sides are worse off if there is no cooperation. The U.N's payoff becomes negative one because no cooperation leads to lower grain exports and higher prices which hurts their efforts to combat food insecurity, through the World Food Programme, along with the welfare of member nations.

The U.N and Russia would benefit the most by exploiting the other. For the U.N., exploiting would mean they do not renegotiate but Russia still decides to cooperate and return to the Black Sea Grain Deal. For Russia, cheating would mean not returning to the Deal when the U.N. makes concessions to renegotiate. For both sides, being exploited leads to the worst payoff.

UN\Russia	C	D
C	3,3	-2,5
D	4,-1	-1,0

Figure 1. Model of the Prisoner's Dilemma stage game used to analyze the Black Sea Grain Initiative

UN\Russia	C	D
C	3,3	-2,5
D	4,-1	-5,-5

Figure 2. Model of the Chicken Game Example

3.2 Model Analysis

The prisoner's dilemma stage game seems appropriate, due to reasons outlined in the previous section, to model the developments surrounding the Black Sea Grain Deal. It best highlights how while both players would benefit more from just mutually cooperating, the incentive to cheat and the fear of being cheating would lead to both sides deciding to not cooperate.

A one-stage strategic game is defined as a game played with no repeated interactions [Rub94]. The prisoner's dilemma stage game from Figure 1 is also a one-stage game. A key attribute of the prisoner's dilemma and other one-stage games are pure strategy Nash equilibria. Pure strategy Nash equilibria are sets of responses where the players cannot make a unilateral deviation that provides a higher payoff. Pure strategy Nash equilibria are strong indicators of the possible final outcomes. This is because they represent when both players follow their respective optimal actions. For example in Figure 1, there is just one pure strategy Nash equilibria under the assumption it is only played for one round: (D,D). Although both sides could mutually cooperate, both choose to defect. Both defect in hopes of either exploiting the other player (U.N and Russia get a higher payoff from (D,C) and (C,D) respectively) and in fear of being cheated (U.N and Russia receives their lowest payoffs from (C,D) and (D,C) respectively). In the prisoner's dilemma, defecting is the U.N and Russia's dominant strategy, meaning it is the action they will take regardless of the other player's response. Once both sides defect, neither player can unilaterally make a profitable deviation, holding the other player's strategy fixed. If the U.N switches from defection to cooperation, their payoff lowers by 1 (-1 v -2), and if Russia is the one that switches from D to C, their payoff is worse (0 vs -1) Therefore, (D,D) is a Nash equilibrium which reflects the realistic outcome given both the U.N and Russia's preferences.

While the prisoner's dilemma game is commonplace in modelling international relations, it is vital to consider alternatives. As aforementioned in [Cor01], the chicken game is a popular alternative stage game to model international crises (p.194). However, there is a key structural component of the chicken game that does not apply to the Black Sea Grain Initiative. In a chicken game (Figure 2), the worst payoffs for both players occur at mutual defection. The pure strategy Nash equlibria become (C,D) and (D,C). The implication of this is that the U.N. and Russia would rather be exploited by the other side then both sides not cooperating. If Russia chooses to defect (drop out of the deal), the U.N would obviously rather not give concessions. However, the equilibria of the chicken game supports the very opposite. In contrast, the prisoner's dilemma stage game reflects the strategic realities for the U.N and Russia. This is especially shown when examining each stage game's equilibira.

First looking at (D, C), the U.N would not want to switch to cooperating and Russia would not want to switch to defection as the payoffs would be worse. The same concept applies to (C, D). As mentioned previously, both pure strategy equilbria of the game support the idea that Russia and the United Nations would rather be taken advantage of than also defecting, which is very unrealistic. So after examining the pure strategy Nash equilibria, the prisoner's dilemma is a more accurate representation and model of how Russia and the U.N would behave compared to the chicken game.

Due to the Black Sea Grain Deal having finite extension lengths, interactions surrounding renegotiation have already occurred multiple times. And with no end to the Ukraine-Russia conflict in the foreseeable future, an infinitely repeated discount game, where the subsequent round's payoffs are discounted by a factor of δ, $\delta \in (0, 1)$, seems to be an appropriate method to analyze the recent tension around the Black Sea Grain Deal.

3.3 Key Definitions

In preparation for my game-theoretic analysis, certain mathematical concepts must be defined. The first of which is an infinitely repeated game. An infinitely repeated game as defined in [Rub94]:

Let strategic game $G = \{N, (A_i), ({}_i)\}$; let A be the set of every player's ($i \in N$) available actions. $A = \times_{i \in N} A_i$; let ${}_i$ be player i's preference relation on A: ${}_i$ on $A = \times_{j \in N} A_j$ where $\times_{j \in N} A_j$ the set of outcomes of A. Applying this to the prisoner's dilemma stage game, Russia and the U.N would be the players of N, and the actions in set A for both Russia and U.N would be cooperate and defect.

An infinitely repeated game of G is an extensive game with perfect information and simultaneous moves $(N, H, P, (^*_i))$. H is the set of histories, which stores the sequence of actions played by all players. This is a fundamental difference between a one-stage game and a repeated game. With one round, there is no prior history to be considered. However, in a repeated game, players will consider all of their previous moves along with everyone else's to inform the action they decide to take, widening the possible number of strategies. Therefore, a history is necessary. In repeated games, P(h) maps a history to a player for each non terminal history $h \in H$ [Rub94]. This means it identifies which player moves and when they move.

While there are variations of an infinitely repeated game involving imperfect monitoring or incomplete information that were considered, I concluded that a model with complete information seemed to be more applicable to the Black Sea

Grain Initiative. This is because the U.N and Russia can clearly observe what the other side is doing. The U.N can tell if Russia decides to cooperate or defect from the Black Sea Grain Deal and Russia can tell if the U.N. has decided to make concessions or not. Thus, it is reasonable that both sides have complete information on the histories of play in the infinitely repeated stage game model.

Utilizing an infinitely repeated game has significant implications for equilibria. Contrary to a game played for only one stage, any mutually beneficial outcome can be supported as an equilibrium when players interact repeatedly. This fact is formally stated in folk theorems [Kan08]. Several folk theorems explore the idea of equilibria in infinitely repeated discounted games. One of the more prevalent theorems is that any individually rational strategy profile can be supported as an equlibrium if δ is close to 1. However, more specific folk theorems have been developed. [GT20] references Fudenberg and Maskin's (1986) folk theorem: "If the number of players is 2 or if the set feasible payoff vectors has non-empty interior, then any payoff vector that is feasible and strictly individually rational is a subgame perfect equilibrium of the discounted repeated game, provided that players are sufficiently patient" [GT20]. Essentially if the players are patient enough, any strictly individually rational strategy can be supported as an equilibrium. Strictly individually rational strategies for any player i are those that yield a higher payoff than player i's min-max strategy [GT20]; the min-max strategy is the payoff a player can guarantee themselves in any equilibrium as explained by [Kan08], which is like the worst-case scenario strategy.

The folk theorem is very broad and lacks predictive power about specific equilibria. It merely suggests that any individually rational strategy could be an equilibrium if the players are patient. Over time, the literature has explored and established more specific folk theorems. [Pea91] explored several of these folk theorems, of which Friedman's (1971) theorem is especially pertinent to the focus of this paper: "Let $G = (A_1..., A_N; \Pi_1, ...\Pi_N)$ have a Nash equilibrium $e = (e_1, ..., e_n) \in A$, and let $q = (q_1, ..., q_n) \in A$ satisfy $\Pi_i(q) > \Pi_i(e)$ for each $i \in N$. Then for δ sufficiently close to 1, there is a sub-game perfect equilibrium of $G^\infty(\delta)$ in which q is played every period on the equilibrium path" (Pearce, 1991). Π denotes the payoff for each player i. Overall, the theorem is very significant as it supports repeated mutual cooperation as a potential subgame perfect equilibrium depending on the players' patience. Similar to the Nash equilibrium in a one stage game, subgame perfect equilibria are strong indicators of final outcomes for infinitely repeated games. So for the infinitely repeated Prisoner's dilemma stage game model, subgame perfect equilibria are key to analyze.

As defined in [Rub94]: "a subgame perfect equilibrium of an extensive game with perfect information $(N, H, P, (^*_i))$ is strategy profile s^* such that for every player $i \in N$ and every nonterminal history $h \in H \backslash Z$ for which $P(h) = i$ we have

$$O_h(s^*_{-i}|_h, s^*_i|_h)_i|_h O_h(s^*_{-i}|_h, s_i|_h)$$

for every strategy s_i of player i in the subgame $T(h)$," (p.97). O_h represents the outcome (the payoff) of a certain strategy profile. What the definition is conveying is that the outcome of player i following the s_i^* strategy is greater than them deviating and following some other strategy s_i holding every other player's strategy s_{-i}^* fixed. Essentially, a strategy profile is a subgame perfect equlibrium if and only if there are no unilateral, profitable deviations in strategy a single player can make. This is why the subgame perfect Nash equilibrium is often referred to as the 'credible threat'. This strongly applies to the Black Sea Grain Initiative because both Russia and the U.N have a threat to defect which would severely punish the other player compared to both sides cooperating.

In an infinite stage game, there are a wide number of possible strategies varying in complexity. Using the aforementioned folk theorems, it is possible for mutual cooperation in every round to be a subgame perfect equilibrium in the infinitely-repeated discounted stage game model. There are several strategies that focus on achieving mutual cooperation: naively cooperating every round or playing tit-for-tat where player i plays the same move their opponent played the round before. However, a common strategy to achieve mutual cooperation is the grim trigger strategy. The aim of a grim-trigger is to use the threat of permanent defection to enforce cooperation. A grim trigger strategy entails always choosing to cooperating until the opposing player defects. Following that, the player using a grim trigger would defect forever, never returning to cooperation. Based on Friedman's folk theorem, it is possible for mutual cooperation, which offers a higher payoff than the one-stage Nash, to be a sub-game perfect. [Sla04] outlines the strategy in a rather eloquent fashion:

$$s_i(h^t) = \begin{cases} C & \text{if } t = 0 \\ C & \text{if } a^\tau = (C, C) \text{ for } \tau = 0, 1, ..., t-1 \\ D & \text{otherwise} \end{cases}$$

4 Results

To find whether mutual cooperation with a grim trigger is a sub-game perfect equilibrium, the payoffs of the strategy and its deviations need to be considered. Using the one-shot deviation principle, as long as there is a single profitable deviation, a strategy cannot be considered sub-game perfect [Rub94].

The payoff of always cooperating for either player would be $\sum_{t=0}^{\infty}(3)\delta^t$ where t increases by 1 with the next stage of the game. For any $\delta \in (0, 1)$, $\sum_{t=0}^{\infty}\delta^t$ yields the discounted sum $\frac{1}{1-\delta}$. The U.N's payoff for cheating is $4+\delta\sum_{t=0}^{\infty}(-1)\delta^t$ Under the grim trigger, the U.N would get a payoff of 4 because Russia would still cooperate while the U.N defects. For future rounds however, Russia would defect forever which means the U.N's optimal response would be to also defect forever (yielding a payoff of -1 which is accordingly discounted by a factor of δ each round). Russia's payoff for cheating is $5 + \delta\sum_{t=0}^{\infty}\delta^t * 0$ which ends up just equalling 5. Under the grim trigger strategy, Russia would get a payoff of 5 because the U.N would still cooperate. For future rounds, the U.N would defect forever, meaning Russia would also do the same as it yields a higher payoff compared to cooperating (0 vs -1). Meaning Russia would receive a payoff of 0 for all future rounds.

In order for the grim trigger strategy, the payoff of always cooperating has to be greater than deviating, cheating one round. This can be represented by the following inequalities:

$$\sum_{t=0}^{\infty}(3)\delta^t \geq 4 + \delta\sum_{t=0}^{\infty}(-1)\delta^t$$

Figure 3. Inequality required to be met to support U.N's grim trigger strategy

$$\sum_{t=0}^{\infty}(3)\delta^t \geq 5 + \delta\sum_{t=0}^{\infty}\delta^t * 0$$

Figure 4. Inequality required to be met to support Russia' grim trigger strategy

Using the discounted sum, the U.N.'s inequality (Figure 3) becomes $\frac{3}{1-\delta} \geq 4 - \frac{\delta}{1-\delta}$. Multiplying both sides by $(1-\delta)$ yields $3 \geq (4-4\delta) - \delta$ which is equivalent to $3 \geq 4 - 5\delta$. Rearranging the inequality yields $5\delta \geq 1$. The solution to the inequality is $\delta \geq \frac{1}{5}$. This means that δ for the U.N must be at least $1/5$ for the grim trigger to be supported. Unless the U.N is extremely impatient with little care for the future, they will follow a grim trigger strategy.

Applying the same process to Russia (Figure 4), the summation on the right side ($\sum_{t=0}^{\infty}\delta^t * 0$) of the inequality simply becomes 0. Multiplying both sides by $(1-\delta)$ becomes $3 \geq 5 - 5\delta$. Rearranging the inequality yields $5\delta \geq 2$. The solution to the inequality is $\delta \geq \frac{2}{5}$ for Russia. Compared to the U.N, Russia requires a higher δ value for the grim trigger strategy to be supported.

Overall, in order for the grim trigger strategy to be a subgame-perfect equilibrium, it needs to be followed by both players. Since Russia has the higher threshold at $\delta \geq \frac{2}{5}$, $\frac{2}{5}$ is the minimum δ value for the grim trigger to be a supported equilibrium. If however δ goes below $\frac{2}{5}$, the grim trigger is not a supported equilibrium as cheating becomes a profitable deviation for Russia.

Based on the findings of the model analysis, the stability of mutual cooperation under a grim trigger depends largely on Russia's patience, which is reflected by the discount factor. As aforementioned, the the δ value could satisfy the U.N's inequality while not satisfying Russia's. The moment the discount factor goes below $\frac{2}{5}$, it is in Russia's best interest to cheat the U.N and permanently drop out of the deal. In my model, the key to understanding the recent developments of the Black Sea Grain Initiative is dissecting how the value of δ can change.

5 Practical Implications

First, it is important to truly understand what δ represents. δ is the factor by which future payoffs are discounted. A higher delta means future payoffs are more valuable when normalised to the value of present payoffs. Extending this idea, δ represents the value placed on the future relative to the present. If δ were to equal 1, that means the future resources/payoffs are equally valuable as those in the present. A δ of 0 implies the future has no value. Since the discount factor represents the value of the future, one should consider the possibility of it varying. This variance can be determined by real world context. Russia has been embroiled in a war with Ukraine for the last year and a half. As a war drags on, a country cares more about the present than the future. The war is causing Russia to divert more present resources, meaning less value in future resources. This explains the discount factor value lowering. This phenomenon has been observed since the deal's inception last July. With each term limit, Russia was gradually more reluctant to extend. This is very apparent during March and May 2023 when Russia only agreed to a 60 day extension, half of the original 120 day extension terms agreed upon. Russia permanently backing out can be explained by their discount factor dropping below the supported threshold, leading to the grim trigger strategy not holding as an equilibrium.

Another factor that could change the δ thresholds to support equilibrium would be change in payoffs. During repeated re-negotiation and as time elapses, payoffs can possibly change [Jer88]. For example, if the payoff for cheating the other player increased, both countries would have a stronger incentive to deviate. Thus, requiring a higher δ to keep them cooperating. An example would be Russia being more incentivized to cheat the U.N and never return to the Black Sea Grain Deal. If Russia's payoff for deviating increased by some value ϵ, where $\epsilon > 0$, the new inequality for Russia to mutually cooperate becomes

$$\sum_{t=0}^{\infty}(3)\delta^t \geq (5+\epsilon) + \delta\sum_{t=0}^{\infty}\delta^t * (0+\epsilon)$$

Using the discounted sum, the inequality becomes $\frac{3}{1-\delta} \geq (5+\epsilon) + \delta(\frac{\epsilon}{1-\delta})$. Multiplying both sides by $(1-\delta)$ yields $3 \geq 5 - 5\delta + \epsilon - \delta\epsilon + \delta\epsilon$. Simplifying, the inequality becomes $5\delta \geq 2+\epsilon$. The solution to the inequality is $\delta \geq \frac{2+\epsilon}{5}$. The minimum delta for the grim trigger to be stable increases by $\frac{\epsilon}{5}$.

Since the U.N's goal is to establish lasting, mutual cooperation for the Black Sea Grain Initiative, they may offer more and more concessions to Russia over time, increasing their payoff for cooperating, reflected by some increase in ϵ This changes the grim-trigger inequality for Russia to become

$$\sum_{t=0}^{\infty}(3+\epsilon)\delta^t \geq 5 + \delta\sum_{t=0}^{\infty}\delta^t * 0.$$

Using the discounted sum, the inequality becomes $\frac{3+\epsilon}{1-\delta} \geq 5$. Multiplying both sides by $(1-\delta)$ yields $3+\epsilon \geq 5-5\delta$. Rearranging yields the inequality $5\delta \geq 2-\epsilon$. The solution to the inequality is $\delta \geq \frac{2-\epsilon}{5}$. By increasing Russia's payoff for cooperation by epsilon, the minimum δ decreases by $\frac{\epsilon}{5}$.

6 Limitations and Possible Extensions of the Game-Theoretic Approach

When utilizing game theory as an analytical tool, there should be a great care and caution. Steven J. Brams addresses this in his 2000 paper. Of the common issues [Bra00] highlight, two apply most to the model outlined in this paper: Misspecifying the rules and confusing the goals with rational choice.

[Bra00] emphasises that the rules outlined in a game-theoretic model should reflect how the players would act in the very situation that's being modeled (p.222). [Bra00] articulates the intuitive idea that the model should reflect how the players in the model would realistically act in the given situation. Another point [Bra00] highlights is that goals and rationality aren't the same. For example, a change in strategy from short to long-term is not varying rationality but rather, it is a variance in goals with the same underlying rationality (p.222). If just a one-stage game was used, it would not realistically affect the decision-making a large country or international governing body (Russia and the U.N) would take. Through the use of an infinite-stage game, Russia and the U.N's long-term lens for decision making is reflected by the model.

The model setup and analysis assumed stable payoffs. While the payoffs of the repeated stage game were discounted by δ, the stage game payoffs themselves remained fixed throughout. As [Jer88] notes, preferences evolve over time. In a constantly changing international environment, preferences, and subsequently payoffs, are likely to change. While this was briefly explored in the Practical Implications by increasing Russia's payoffs to defect and cooperate by some positive epsilon, more research needs to be done to exactly quantify the payoffs and how they evolve over time.

While the infinitely repeated Prisoner's dilemma provides a relatively comprehensive analysis of the current developments surrounding the Black Sea Grain Initiative, future developments may require an adjustment or rethinking of the model. Even now, there may be opportunities to expand and advance the current model. While doing so, it is important to remember what the core purpose of game-theoretic models is in international relations.. They are supposed to provide structure that aligns with players' realistic thinking and actions which can be analyzed and studied. Adding or reinventing the model should only be done after extensive and thorough consideration.

7 Conclusion

This paper provided a game-theoretic analysis on the Black Sea Grain Initiative using an infinitely repeated Prisoner's Dilemma stage game. It is possible to structure a model such that cooperation primarily depends on Russia's patience. The war in Ukraine carrying on for over a year and a half has decreased Russia's valuation of the future. Slight increases in the payoff to defect can increase the minimum δ for Russia to cooperate. However, the U.N can offer more concessions to increase Russia's payoff to keep cooperating, therefore decreasing the minimum δ required. These findings outline the cause of the breakdown: a lack of patience on Russia's behalf. This lack of patience (a low valuation of the future)makes Russia unwilling to extend the Deal as it is not as beneficial to them. However, the findings presented also find a solution to preserve cooperation around the Black Sea Grain Deal: offering more concessions to Russia to incentivize a return to the Black Sea Grain Deal. However, the U.N has to offer enough concessions that renewing the Black Sea Grain Deal is beneficial to Russia, even with a lower patience, in order for Russia to cooperate.

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