HIGH SCHOOL EDITION
Journal of Student Research
Volume 14 Issue 1 (2025)

Analyzing Methods of Optimizing Low ΔV Gravity Assist Trajectories in the Jovian System

Adi S. Jha[1] and William Brandenburg[#]

[1]West Windsor Plainsboro High School North, USA
[#]Advisor

ABSTRACT

To accomplish scientific objectives, spacecraft exploring the Jovian system require close flybys of Jupiter's inner moon of Europa. However, the ΔV cost of a direct transfer to Europa is prohibitively expensive, so ballistic flybys with the other moons in the system are required to lower the energy of the spacecraft relative to Jupiter. Sequences of ballistic flybys can simultaneously alter the spacecraft's trajectory in a desirable way and provide additional opportunities for scientific observation. Using a patched conics simulation, the N-Revolution Lambert's problem can be solved to find sequences of flybys that enable low ΔV transfers between the four major Jovian moons.

Introduction

Background

Spacecraft that explore the outer solar system tackle the problem of high ΔV costs in the transfer to the outer body and the capture burn. While including more fuel on on a spacecraft increases its possible ΔV, this comes with several drawbacks. If the satellite's mass is increased due to extra fuel being added, the cost to launch is increased due to a larger payload. If the mass fraction, the ratio of wet to dry mass, of the satellite is increased, the scientific potential of the satellite will be decreased due to less dry mass being available. Due to these drawbacks, many spacecraft have employed the use of gravity assists to alter their orbit without the use of extra fuel .

Famous examples of these spacecraft include the Galileo and Cassini spacecraft.

The Gallileo spacecraft, the first spacecraft to orbit Jupiter, employed gravity assists during many points in its mission. Before its mission at Jupiter started, it used the gravity of Venus and Earth, flying by the former once and the latter twice, to set it on a trans-Jovian transfer orbit. In addition to these gravity assists, it employed the gravity of Io to assist it in a Jovian capture, allowing it to save several kilometers per second of ΔV.

The Cassini spacecraft, the first spacecraft to orbit Saturn, also employed gravity assists during many points in its mission. Flying by Venus twice, Earth once, and Jupiter once, the spacecraft was able to save incredible amounts of ΔV in its transfer to Saturn.

While at their respective planets, both the Gallileo and Cassini spacecraft used velocity changes from gravity assists off one moon to reach another moon, enabling them to gather scientific data on several moons without spending large amounts of ΔV on transfer orbits. These chains of successive gravity assists will be used in future missions, such as the Europa Clipper, to gather large amounts of data on certain moons. Thus, further research on these gravity assist trajectories will be essential to enabling other complex missions in the outer solar system in the near future.

Problem Statement

To maximize the scientific output of a mission at Jupiter, flybys of several moons should ideally be completed as successive flybys will allow the spacecraft to collect larger amounts of scientific data compared to spacecraft that only complete singular flybys. The objective of this study is to find a trajectory that flies by Europa while also minimizing the ΔV expended by the spacecraft, keeping it under 2 kilometers per second.

Theoretical Framework

Methods of Orbital Simulation

There exists many ways of simulating orbital systems; however, this work will focus on two methods: n-body simulations and patched conic models .

N-Body Simulations

In n-body simulations, the gravitational interactions between multiple bodies in an orbital system are considered simultaneously, resulting in multiple acceleration vectors acting on each body while increasing accuracy in modeling the system. The net acceleration vector of body i in an n-body system is given by the equation:

$$\mathbf{a}_i = \sum_{j=1}^{n} G \frac{m_j(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3}, \quad j \neq i$$

where r_i and r_j represent the position vectors of bodies i and j, m_j represents the mass of body j, and G is the universal gravitational constant.

The motion of each body follows Newton's second law of motion combined with the law of universal gravitation shown above. Thus, the position $\mathrm{r}_i(t)$ of body i varies according to:

$$\mathrm{F}_i = m_i \mathrm{a}_i = m_i \frac{d^2 \mathrm{r}_i}{dt^2}$$

By substituting the expression for a_i, the second-order differential equation determining the motion of body i is obtained:

$$m_i \frac{d^2 \mathrm{r}_i}{dt^2} = \sum_{j=1, j\neq i}^{n} G \frac{m_i m_j(\mathrm{r}_j - \mathrm{r}_i)}{|\mathrm{r}_j - \mathrm{r}_i|^3}$$

Simplifying:

$$\frac{d^2 \mathrm{r}_i}{dt^2} = \sum_{j=1, j\neq i}^{n} G \frac{m_j(\mathrm{r}_j - \mathrm{r}_i)}{|\mathrm{r}_j - \mathrm{r}_i|^3}$$

The above second-order differential equation describes both the acceleration and the motion of body i under the gravitational influence of all the other bodies in the system. To solve for the position of a particular body at time t_f, the positions for all n bodies must be solved simultaneously from time t_0, the initial condition of the system, to time t_f. Due to there being more differential equations than constants of motion, no analytical solution exists for the

n bodies. Instead, numerical integration techniques must be employed to approximate the trajectories of the bodies in the system.

A commonly used numerical integration technique is the Runge-Kutta method, specifically the fourth-order method (RK4). Compared to a more basic method, such as the Euler step method, the error scales as h^4 while the error in the Euler step method scales as h. This means larger step sizes can be taken with the RK4 method while maintaining a similar level of accuracy.

The RK4 method approximates the solution of ordinary differential equations (ODEs) by using a weighted average of the function's slopes, evaluated at different points within each step. For a first-order ODE of the form:

$$\frac{dy}{dt} = f(t, y)$$

RK4 calculates the value of $y(t + \Delta t)$ at each time step by computing four intermediate slopes:

$$k_1 = f(t_n, y_n)$$
$$k_2 = f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} k_1\right)$$
$$k_3 = f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} k_2\right)$$
$$k_4 = f(t_n + \Delta t, y_n + \Delta t\, k_3)$$

where:

- k_1 represents the slope at the start of the step,
- k_2 and k_3 represent mid-step slopes, incorporating information from the earlier slopes,
- k_4 is the slope at the end of the step.

The updated value of y_{n+1} is then determined using a weighted average of these slopes:

$$y_{n+1} = y_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Although the equation of motion described earlier is a second-order differential equation, it is possible to rewrite it as a first-order ODE. For each body, the position and velocity vectors are treated as separate variables, splitting the original second-order differential equation into:

$$\frac{d\mathrm{r}_i}{dt} = \mathrm{v}_i$$
$$\frac{d\mathrm{v}_i}{dt} = \mathrm{a}_i$$

where a_i is the gravitational acceleration. The RK4 method is applied to these first-order equations to update both position and velocity at each time step. Thus, the position of each of the n-bodies can be numerically solved for.

Patched Conic Approximations

Patched conic approximations are a simplified method of modeling the trajectories of spacecraft under the gravitational influence of multiple bodies. Unlike n-body simulations however, which account for the simultaneous gravitational interactions between all bodies, patched conic simulations divide the spacecraft's trajectory into separate segments. Each of these segments only consider the gravitational influence of a singular body, significantly reducing computational complexity.

Since the patched conic method only considers the gravitational attraction between the spacecraft and a singular body, a 2-body system, the analytical solution to the system can be found and used to determine the location of a particular moon or a spacecraft.

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

To find the rate of phase angle change, or mean motion ν, divide 2π by the expression for T:

$$\nu = \frac{2\pi}{T}$$

Substituting the expression for T:

$$\nu = \frac{2\pi}{2\pi\sqrt{\frac{a^3}{\mu}}}$$

Simplifying:

$$\nu = \sqrt{\frac{\mu}{a^3}}$$

The relationship between mean anomaly M and eccentric anomaly E is given by Kepler's equation:

$$M = E - e\sin E$$

This equation must be solved numerically to find E.
Once E is found, the true anomaly θ is computed using the relation:

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$

Alternatively, using sine and cosine relations:

$$\cos\theta = \frac{\cos E - e}{1 - e\cos E}$$

$$\sin\theta = \frac{\sqrt{1-e^2}\sin E}{1 - e\cos E}$$

The radial distance r from the focus of the orbit is given by:

$$r = a(1 - e\cos E)$$

where a is the semi-major axis, and e is the eccentricity.

To convert the position from the orbital plane to 3D space, apply three rotations using the orbital elements: the inclination i, the longitude of the ascending node Ω, and the argument of periapsis ω.

The transformation to 3D coordinates is given by:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix} \begin{pmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ 0 \end{pmatrix}$$

$$x = r((\cos\Omega\cos(\omega+\theta) - \sin\Omega\sin(\omega+\theta)\cos i))$$

$$y = r((\sin\Omega\cos(\omega+\theta) + \cos\Omega\sin(\omega+\theta)\cos i))$$

$$z = r(\sin(\omega+\theta)\sin i)$$

Thus, the position of a body at a certain true anomaly θ can be analytically solved for.

Gravity Assist Mechanics

Sufficiently close intersections of the two orbits allows for a momentum transfer between the two bodies, altering the orbit of both bodies. When a spacecraft's orbit intersects a moons orbit, a gravitationally assisted flyby occurs. As the spacecraft's mass is orders of magnitude smaller than any of the Jovian moons, the moons orbits remain unchanged while the spacecraft's orbit is altered significantly. This can be proved using the law of conservation of momentum and energy.

Conservation of Energy

Consider a system with three bodies. Body A is the central body; body B is a body orbiting body A; body C is a spacecraft flying by body B. The spacecraft is first considered in the frame of reference of the body B. With respect to body B, the spacecraft is travelling with some velocity $v_{\infty+}$ at a radius of r. Thus, its energy at any point can be described using the equation:

$$U = \frac{1}{2}m_s v^2 - \frac{Gm_b m_s}{r}$$

Where m_s is the spacecraft mass, m_b is the mass of the body B, and v is the spacecraft's velocity.

Before the assist, the energy is:

$$U_i = \frac{1}{2}m_s v_{\infty+}^2 - \frac{Gm_b m_s}{r}$$

After the assist, it becomes:

$$U_f = \frac{1}{2}m_s v_{\infty-}^2 - \frac{Gm_b m_s}{r}$$

Before the gravitational assist, the spacecraft can be considered to be at a distance r, infinitely far away from body B. This consideration simplifies the gravitational potential energy to 0 in both equations.

$$U_i = \frac{1}{2}m_s v_{\infty+}^2$$

$$U_f = \frac{1}{2}m_s v_{\infty-}^2$$

Since the spacecraft is being considered in the frame of reference of body B, the problem becomes a localized two-body problem. In conjunction with body B's mass being much larger than the spacecraft, mitigating its velocity change, the initial and final energy of the system can be considered as an elastic gravitational collision. Thus, the initial and final energy of the system are equivalent, producing:

$$\frac{1}{2}m_s v_{\infty+}^2 = \frac{1}{2}m_s v_{\infty-}^2$$

Thus, $v_{\infty+} = v_{\infty-}$, meaning the hyperbolic excess velocity remains constant in magnitude before and after the assist; however, this needs to be converted into the frame of reference of body A.

Proof of Velocity Change Relative to Body A

Let the spacecraft's initial velocity relative to body A be $\mathrm{v_i}$, and let $\mathrm{v_b}$ be the velocity of body B relative to body A. The spacecraft's initial hyperbolic excess velocity relative to the assisting body before the flyby is:

$$\mathrm{v}_{\infty-} = \mathrm{v_i} - \mathrm{v_b}$$

After the flyby, the magnitude of the hyperbolic excess velocity remains constant, but the direction is altered by the turn angle δ:

$$\mathrm{v}_{\infty+} = R(\delta)\mathrm{v}_{\infty-}$$

where $R(\delta)$ is the rotation matrix that rotates vectors by angle δ in the plane of the flyby.

The final velocity of the spacecraft relative to the central body is:

$$\mathrm{v_f} = \mathrm{v_b} + \mathrm{v}_{\infty+}$$

To find the change in the magnitude of the spacecraft's velocity relative to body A, we compute the squares of the initial and final velocities:

$$|\mathrm{v_i}|^2 = |\mathrm{v_b} + \mathrm{v}_{\infty-}|^2 = v_b^2 + v_\infty^2 + 2v_b v_\infty \cos\theta_i$$

$$|\mathrm{v_f}|^2 = |\mathrm{v_b} + \mathrm{v}_{\infty+}|^2 = v_b^2 + v_\infty^2 + 2v_b v_\infty \cos(\theta_i + \delta)$$

where θ_i is the angle between $\mathrm{v_b}$ and $\mathrm{v}_{\infty-}$.

The change in the square of the velocity magnitude is then:

$$|\mathrm{v_f}|^2 - |\mathrm{v_i}|^2 = 2v_b v_\infty[\cos(\theta_i + \delta) - \cos\theta_i]$$

Using the trigonometric identity:

$$\cos(\theta_i + \delta) - \cos\theta_i = -2\sin\left(\frac{\delta}{2}\right)\sin\left(\theta_i + \frac{\delta}{2}\right)$$

Obtain:

$$|\mathrm{v_f}|^2 - |\mathrm{v_i}|^2 = -4v_b v_\infty \sin\left(\frac{\delta}{2}\right)\sin\left(\theta_i + \frac{\delta}{2}\right)$$

This equation shows that the change in the spacecraft's velocity magnitude depends on both the turn angle δ and the initial angle θ_i. Depending on the values of these angles, the spacecraft's speed relative to the central body can increase, decrease, or remain the same as a result of the gravity assist.

Final Velocity Vector

To find the final velocity vector $\mathrm{v_f}$, we first compute the post-flyby hyperbolic excess velocity relative to the assisting body:

$$\mathrm{v}_{\infty+} = R(\delta)\mathrm{v}_{\infty-}$$

Then, adding the velocity of the assisting body:

$$\mathrm{v_f} = \mathrm{v_b} + \mathrm{v}_{\infty+}$$

The magnitude of $\mathrm{v_f}$ is:

$$|\mathrm{v_f}| = \sqrt{v_b^2 + v_\infty^2 + 2v_b v_\infty \cos(\theta_i + \delta)}$$

Lambert's Problem and its Solutions

Lambert's problem is a classical problem in patched conics astrophysics. Given an initial position vector $\mathrm{r_1}$, a final position vector $\mathrm{r_2}$, a time of flight t, and a gravitational parameter μ, two elliptical arcs can be traced between the vectors. These elliptical arcs correspond to the patched conic orbit that contains both of these position vectors.

While analytical methods of solving Lambert's problem exist, this paper uses the solution written by R.P. Russel. This solution returns the velocity vector needed at $\mathrm{r_1}$ to reach $\mathrm{r_2}$, the velocity vector at $\mathrm{r_2}$ from $-N$ to N revolutions .

Solution Method

Initial Conditions and Setup

Before initializing the simulation, a method of orbital simulation must be chosen. While an n-body method of simulation would yield the highest accuracy, its high computation time discourages its use; instead, a patched conics system is used to simulate the spacecraft. This is then coupled with a higher accuracy method of obtaining the position of the moons of Jupiter.

Ephemeris data from the SPICE toolkit is used instead of directly simulating the motion of the moons. SPICE is an ancillary information system used to generate ephemeris data through n-body simulations at high levels of fidelity .

Thus, instead of simulating the motion of each moon until a certain time t is reached, the SPICE toolkit can be called at the time t, providing the position of each moon quickly.

Now that a method of simulation has been determined, the initial conditions of the system need to be defined. The spacecraft is started in a circular orbit with parameters displayed in Table 1. To obtain the initial conditions of the moons, an initial time is chosen and the location of each moon is called from the SPICE toolkit. With the initial time being January 11th, 2026, 1:00:00 UTC, the position and orbital characteristics of each is found. These values are provided in Table 2.

Obtaining Transfer Orbits

Callisto Transfer Orbit

Due to the semi-major axis of Callisto being the closest to the initial semi-major axis of the spacecraft, it becomes an obvious first target in the trajectory.

While a simple Hohmann transfer orbit could suffice as a transfer orbit to Callisto, to increase the magnitude of the relative velocity of the spacecraft, the time of flight of the transfer is slightly reduced. Once the time of flight is obtained, a simple difference in mean motion calculation is used to find the ideal time of impulse and the Lambert's problem solver is called to find the velocity needed to complete the transfer.

The flyby of Callisto can have many possible geometries based on the turn angle; however, all of these geometries do not need to be found. Instead, the range of geometries is reduced to the semi-major axis after the flyby, which can be easily calculated with the velocity after flyby and the position of the spacecraft at closest approach. The upper limit of the range is arbitrarily set as the semi-major axis after a flyby 5000 kilometers from the center of Callisto. While no change would occur at a distance infinitely far away from Callisto, this flyby would not provide any information on the trajectory after flyby.

On the other hand, the lower limit of the range is defined as the semi-major axis after a flyby 5 kilometers from the surface of Callisto. The closeness of this flyby would maximize the gravitational pull between Callisto and the spacecraft, maximizing the change in the spacecraft's orbit. While closer approaches are possible, they risk a collision between the spacecraft and Callisto; therefore, a 5 kilometer flyby will minimize this risk while maximizing the change in semi-major axis.

The semi-major axes for the limits of the range are obtained along with the velocity relative to Jupiter after flyby, allowing for the other orbital parameters to be found. Although the range of values is known, since the objective of the flyby is to lower the spacecraft's orbit such that it can encounter Ganymede, the lower limit of the range is selected.

The new orbital parameters are then applied and the simulation continues.

Ganymede and Europa Transfer Orbits

Once the new orbital parameters have been applied, a transfer orbit from the spacecraft to Ganymede needs to be calculated. This new transfer is much harder to obtain as one of the initial orbits is elliptical; thus, a brute force method has to be used to find the ideal transfer orbit.

By varying the time of flight, t, of the transfer orbit by some arbitrary factor a, non-hohmann multi revolution transfer orbits can be obtained. To find the velocity needed for these non-hohmann transfers, the Lambert's problem solver can be called. The arguments that are then passed are:

- The initial position vector, $\mathbf{r_1}$
- The final position vector, which is simply Ganymede's position after t, $\mathbf{r_2}$
- The time of flight, t
- The number of rotations, N, which is dependent on the time of flight

The needed velocity is then compared with the current velocity and selected if its magnitude is under a certain threshold.

Once a transfer orbit has been found, the ΔV is applied and the spacecraft makes its way to Ganymede, the velocity change due to a flyby is found, and the process is repeated to find a transfer orbit to Europa.

Results

After running the simulation, a trajectory that flies by Callisto, Ganymede, and Europa was found. This trajectory takes approximately seconds, which is approximately 86.947 days, while only using approximately of ΔV. The spacecraft's trajectory overlaid with the orbits of each moon is shown below.

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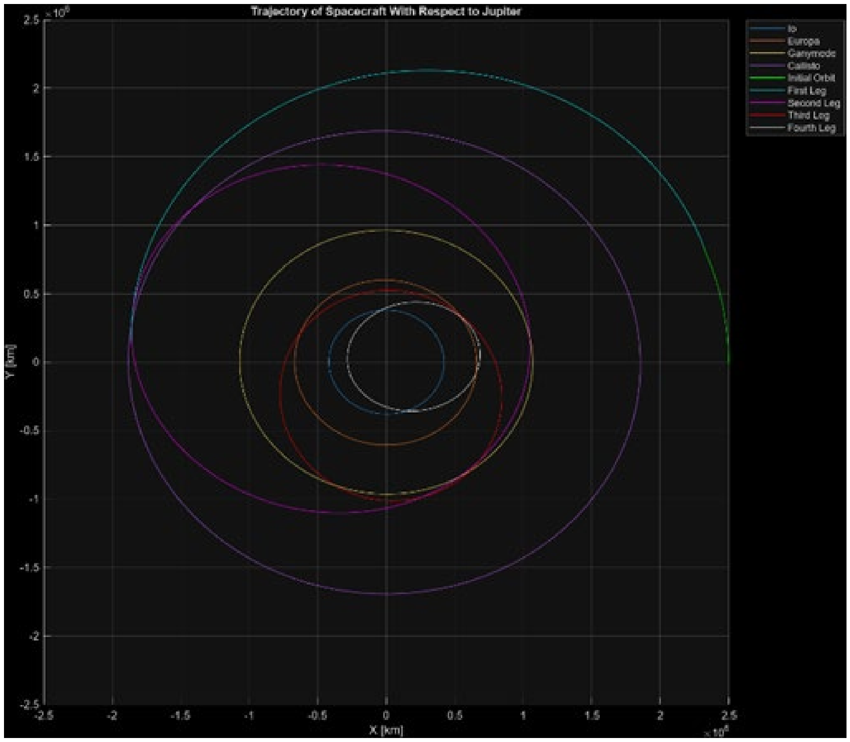

Figure 1. Trajectory of Spacecraft Overlaid with Orbits of Jupiter's Moons

The spacecraft's trajectory on its own is displayed below with impulses and flybys shown. The first impulse is given by the red dot; the flyby of Callisto and the second spacecraft impulse is given by the orange dot; the flyby of Ganymede and the third spacecraft impulse is given by the yellow dot; and the flyby of Europa is given by the green dot.

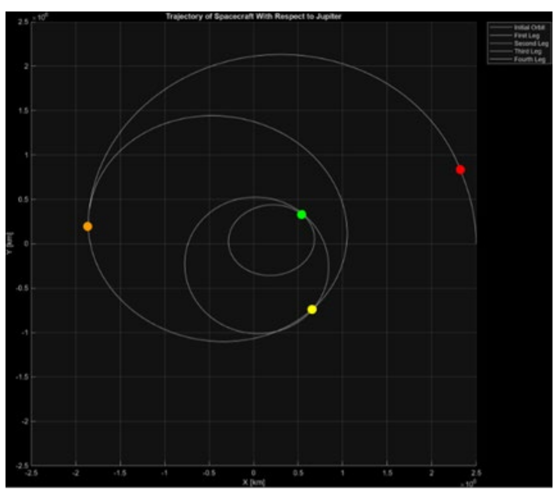

Figure 2. Standalone Trajectory of Spacecraft Overlaid with Impulses

The spacecraft, as stated in the solution method, starts at distance of from the center of Jupiter. After travelling for a short period of time, it applies a ΔV of at the position indicated by the red dot to encounter Callisto at the position indicated by the orange dot. Once at Callisto, it flies by at an altitude of 5000 meters, slowing its orbital velocity. This new trajectory however, will not encounter Ganymede in a reasonable amount of time. Thus, a ΔV of is also given during the flyby so the spacecraft can encounter Ganymede at the position indicated by the yellow dot.

After orbiting Jupiter for approximately 50.41 days, the spacecraft encounters Ganymede at an altitude of 5000 meters, slowing its orbital velocity once again. The new trajectory after flyby does intersect the orbit of Europa; however, improper phasing between the two moons requires the spacecraft to apply a ΔV of to properly encounter Europa at the position indicated by the green dot.

After orbiting Jupiter again for approximately 26.28 days, the spacecraft encounters Europa at an altitude of 5000 meters, slowing its orbital velocity for the final time. This final orbit dips close to Jupiter, with a perijove of 3.027×10^8. This low orbit enables the spacecraft to be flown into Jupiter to prevent contamination of the Jovian moons after its mission is over.

While the spacecraft needed a significant amount of external ΔV to complete this tour of the Jovian system, the flybys of each of the moons contributed a larger change in velocity, illustrating their effectiveness in changing orbits. During the flybys of the Jovian moons, the spacecraft lost an additional of ΔV solely from the gravity assist off Callisto; of ΔV solely from the gravity assist off Ganymede; and of ΔV solely from the gravity assist off Europa. Adding the total impulse given by the Jovian moons, a result of is obtained, illustrating the effectiveness of gravity assist trajectories in this tour.

Discussion and Conclusion

Effectiveness of Gravity Assists

As shown in the prior section, the flybys of the various Jovian moons decreased the required ΔV from the spacecraft significantly and the tour of the Jovian system was completed within the ΔV limitation and in a reasonable time frame. The strategic planning of the gravity assists off Callisto, Ganymede, and Europa enabled the spacecraft to minimize the transfer cost between each moon, leading to the overall tour's ΔV cost being minimized.

The choice of each of the moons was also significant and beneficial to the tour's success. Due to the spacecraft starting at a distant orbit, Callisto was a clear first target because of its orbital proximity to the spacecraft. Due to this proximity in orbits, the spacecraft did not have to expend much ΔV in the Callisto transfer. If the first transfer was with a moon with an orbit closer to Jupiter, the transfer ΔV cost would have been much higher, potentially ruling out the trajectory due to a great ΔV cost. The subsequent choice of Ganymede as the second flyby location reiterates the logic before, minimizing inter-lunar transfer costs to minimize the overall trajectory ΔV cost.

Accuracy and Limitations

While the simulation was able to obtain a possible trajectory a potential spacecraft could go through in the Jovian system, there exist several factors that could limit the feasibility of the tour in the real world.

Orbital Model

Although the orbital model for the Jovian moons uses a high-fidelity n-body simulation, the position of the spacecraft uses a patched conics approximation. Due to the several moons in the Jovian system and the elliptical nature of the transfer orbits, a patched conics approximation will have some error compared to the actual position of the spacecraft. These perturbations in the spacecraft's orbit may necessitate correction burns throughout the trajectory, increasing the overall ΔV cost of the tour.

The simplification of the gravitational model also gives rise to the issue of unequal mass distribution of Jupiter and its moons, further complicating the differential equations of motion of each of the bodies in the simulation.

Other forces, such as solar wind and interplanetary dust, also have the possibility of influencing the trajectory of the spacecraft significantly, requiring them to be simulated for increased accuracy.

ΔV Simplifications

Throughout the simulation, the various changes in orbital velocity were modeled as instantaneous changes in velocity. While this simplification reduced complexity, it decreased the accuracy of the projected trajectory of the spacecraft. Impulses from spacecraft take time to complete as rocket engines have a certain mass flow rate. While shorter burns can be approximated as instantaneous impulses, longer burns, such as the ones present in this tour, will experience trajectory drift as a result of this simplification. In addition to the ΔV changes from the spacecraft, gravity assists were modeled as instantaneous changes in velocity, a simplification of the actual hyperbolic trajectory the spacecraft goes through.

Future Work

Although this work considers several simplifications of orbital dynamics, these findings illustrate the several directions of potential future work. Including a more complex and thorough orbital model will mitigate the errors with the patched conics approximation in this work, greatly increasing the accuracy of the predicted trajectory.

Another area of improvement is the number of flybys completed by the spacecraft during its tour of the Jovian system. While the interplanetary transfer time between the Earth and Jupiter takes several years, the Jovian tour itself takes a fraction of the time. Increasing the time spent at Jupiter increases the time of scientific research by allowing the spacecraft to collect more data. This can be accomplished with multiple flybys of the same moon through resonant orbits, allowing the spacecraft to collect data from each moon over longer periods of time.

Although the spacecraft saved multiple kilometers per second of ΔV, more could be saved with proper phasing between each of the moons. While the position of each moon cannot be changed on a whim, repeated flybys of moons enables greater change in the spacecraft's orbit. Careful tuning of these repeated flybys could lower or even eliminate the need for the spacecraft to provide an impulse during the flyby, increasing the mission's efficiency.

Acknowledgments

I would like to thank my advisor for the valuable insight provided to me on this topic.

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