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ABSTRACT

As literature examining the application of game theoretic models to political coalitions has flourished in the past few decades, the conclusions found by different types of models yield different predictions regarding payoff proportionality in parliamentary coalitions. While demand bargaining models predict pure payoff proportionality, proposal-based models predict that the formateur should receive a disproportionately large share of the total payoff—a phenomenon referred to as the formateur’s advantage. We apply a statistical model that incorporates this idea to calculate the predicted share of cabinet posts received by member parties of Israeli Governments 25–35, and then compare these predictions to each party’s actual share of cabinet posts in each government. We find a very strong correlation (r = .936) between the predicted and actual shares of cabinet posts for all parties, but find that the formateur’s advantage, while present, is overestimated by bargaining models, as the extent of the advantage is heavily determined by the context in which governments are formed. We then cross-analyze discrepancies in our data with the real-world factors that caused them in order to present evidence that empirical context should always be accounted for when applying game theoretic models to parliamentary coalition formation.

Introduction

A coalition forms when two or more people come together to make decisions, and a coalition government is one in which two or more parties must pool their votes to gain legislative power, with the goal of creating a majority where one did not previously exist (Felsenthal, 1979). Political coalitions are vital in ensuring that a democracy runs effectively and that different political parties are able to compromise on legislation. Coalition theory, which seeks to understand how coalitions form and function, was popularized after the establishment of Gamson’s Law, which predicts a strictly proportional relationship between a party’s share of seats in parliament and share of cabinet posts in the country’s cabinet, or executive branch. Since, myriad studies that examine the law’s theoretical and empirical ramifications have contributed invaluably to the renaissance of coalesional game theory¹. Particularly, Gamson’s Law has served as a profound, puzzling contribution to the study of noncooperative bargaining games², pointing to a mismatch between theoretical expectations and

¹ Game Theory: the process of modeling strategic interactions between two or more people in a situation containing rules and outcomes.
² Any game scenario that does not permit enforceable agreements, where bargaining occurs. In this case, non-enforceable agreements are made among potential coalition partners, who work to maximize their own benefit, and bargaining must occur in order to form a coalition.
empirical patterns; while game theory expects payoffs to correspond to bargaining weights, Gamson notices that, empirically, payoffs correspond to seat weights.

Noncooperative bargaining games use Gamson’s Law of Proportionality as a frame of reference when studying the allocation of resources within a government—in our case, cabinet posts, which are positions in a country’s cabinet held by a person, not specific to a certain jurisdiction. Some bargaining games attempt to predict the ex-post payoff of any particular party in a system by creating models incorporating bargaining theories, often using coalition formation and cabinet post allocation as empirical applications. It must be acknowledged, however, that the majority of coalitional studies—empirical and theoretical studies alike—include the same 14 Western European parliamentary democracies. Testing of these games and their application to real governments has been, for the most part, limited to these countries. This greatly limits the extent to which we can analyze the empirical foundations of game-theoretic concepts.

The Israeli government provides a unique opportunity to test the conjectures of game theories. Multiple coalition studies of the Israeli government have been conducted; however, they were written in a different age, when power in the Israeli government was distributed far differently than today, and when coalition theory was still in its infancy (Nachmias 1974; Mahler 1975). Since then, coalition theories have become greatly sophisticated. Due to its unique nature—defined by strong identity-based divides—we use the Israeli government to accomplish the following: test the conjectures of bargaining theories, discuss how context factors like interpersonal relationships and identity-based difference affect the coalition formation process, and revisit Nachmias and Mahler’s works to evaluate how coalition politics have changed in the Israeli government since the 1970s.

Fundamental Coalition Theory

The Birth of Game Theory

To understand the developments in the field of political coalition theory, it is necessary to first discuss the first generation of game theorists. All studies of coalition theory stem from Von Neumann and Morgenstern’s *Theory of Games and Economic Behavior* (1944), which predicted that a legislative political coalition would have no unnecessary partners, which means that every member party (MP) must be absolutely necessary in forming a winning coalition—such that the subtraction of any MP would result in a losing coalition—since the winning parties will not want to share their spoils with more partners than necessary (1944). This idea has manifested itself as the “zero-sum condition”, which identifies coalition formation in government as a “zero-sum game.”

William Riker’s work is paramount in its application of bargaining theory to political coalitions (1962). He models the formation of coalitions as a fixed-sum bargaining game, in which participants must agree how to divide something universally valuable (government membership in this case), agreeing upon who joins the coalition and under what conditions. Riker builds off Von Neumann and Morgenstern’s zero-
sum condition, establishing the essential “minimal winning coalition” ($W_m$), which states that only the minimal winning coalition with the smallest number of MPs will form. In a minimal winning coalition, the subtraction of any single member would result in a “non-winning” coalition. Since government membership is considered a valuable prize, this coalition type should be favored. However, since information is not always perfect\(^8\), Riker also suggests that oversized coalitions, which are slightly larger than minimal winning size, may form. This idea has manifested itself into the “minimal-size-plus” theory, which has been operationalized a number of ways. In a study of coalition governments in Western Europe, Mitchell and Nyblade (2006) found that roughly two-fifths of majority coalitions were oversized, which means that the coalition has more members than absolutely necessary. Leiserson (1968) proposes a further specification to the minimal winning condition, arguing that coalitions with the fewest number of parties are most likely to form and are easiest to maintain. This study will later see this particular theory manifesting itself in the Israeli case.

**Payoff Proportionality**

Gamson (1961) proposes alternative conditions to Riker’s theories, which differ in multiple regards. While Riker assumes the zero-sum condition of coalition formation—where no party will be invited to join a coalition if their seats aren’t needed to create a majority bloc—Gamson does not. However, Gamson also predicts that payoffs to parties must be proportional to the resources (typically in the form of seats in parliament that contribute to a majority bloc) that they contribute to the coalition (382). Axelrod (1970) adds yet another dimension to coalition theory, predicting that parties that are closer to each other on a policy continuum will be most likely to form, predicting that a “conflict of interest” (perhaps caused by ethnic or theological differences) between parties that are ideologically distanced will prevent seamless coalition formation. He predicts that “minimal-connected winning coalitions” are most likely to form and survive for a longer duration of time (98).

Gamson’s proportionality theory was first empirically tested by Browne and Franklin, who tested the hypothesis that a party’s share of seats in parliament would be directly proportional to their share of cabinet posts. Their analysis found a staggering correlation coefficient of .926, indicating that the relationship was proportional on a nearly one-to-one basis, cemented Gamson’s law of proportionality as one of the strongest relationships in the social sciences. This means that theoretically, a party whose seats comprise 10% of a parliament should receive 10% of the cabinetry as a reward for joining a winning coalition.

**Developments in Coalition Theory**

Since the appearance of previously referenced fundamental works, much progress has been made to move beyond the conclusions previously discussed. The bargaining environment, the distribution of bargaining power, and specific government membership are three crucial elements of coalition theory that have evolved within the scholarship; a review of this literature is necessary in order to understand the unique nature of the Israeli case.

**Bargaining Environment**

While Riker’s work was fundamentally justified in its conclusions, it often fails to predict and rationalize divergences (instances which seem to defy previous theories). The bargaining environment, defined as the type of environment in which coalitions form, is a critical element of coalition formation, which classic scholarship did not recognize. This includes the type of parliament coalitions form in, the nature of political parties in a parliament, and institutional rules that guide coalition formation. Whether or not a country requires a

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\(^8\) Perfect information: all information is known to all parties (ie. chess, where all piece positions are universally known).
majority coalition to be formed, the additional requirement of a supermajority, and a potential vote of no confidence are all examples of institutional guidelines that create a unique bargaining environment. Riker’s work was based on models of cooperative game theory, which assumes that the most “efficient” governments will form. However, a substantial body of research indicates that non-cooperative models, which focus on the incentives of individual parties, better capture real-world bargaining dynamics. Also, they allow us to better understand the importance of institutional rules that impinge on coalition bargaining. The requirements that political institutions place on coalition formation greatly shape the environment in which parties bargain (Strom and Nyblade, 2009). Laver and Shepsle argue that the set of cabinet portfolios and their respective jurisdictions also restrict the feasible policy options on which a coalition could agree, since it would be difficult to prevent the holder of any one portfolio from implementing their preferred policy in their jurisdiction (1996). For example, if an ultra-religious party controlled the Ministry of Religious Affairs, it would be difficult to implement policies that promote secularism. These restrictions set by available portfolios are further explored by Warwick and Druckman, who indicate that since certain portfolios are relatively more important than others, and that each should have a specific weight depending on their importance (2001). A party whose primary platform is national security holds the Ministry of Defense to a higher value than the Ministry of Education, which a party more concerned with the welfare of citizens may hold to a higher value than the Ministry of Defense. They also found that the formateur tends to receive a disproportionately large share of the total payoff. This idea is formalized in theory by the formateur’s advantage, which we will discuss later.

**Government Type**

Theoretical specifications have allowed us to predict which type of coalition is most likely to form under certain conditions (minority, majority, and minimal-winning coalitions). In particular, the distribution of bargaining power among parties has a notably large impact on coalition size and payoff distribution. Bargaining power is often measured in terms of minimal-integer voting weight, as we will discuss further on. The logic of non-cooperative models (Laver and Shepsle, 1996) suggest that a minimal winning coalition should form when bargaining power is neither greatly concentrated in the hands of one party, nor dispersed so that a surplus government is formed. This condition is often present in empirical analyses of the Knesset and other legislatures. Conversely, a consistent finding in coalition studies suggests that near-majority parties are likely to form minority governments, where the MP’s control less than half of the seats in parliament. Mitchell and Nyblade (2006) suggest that when the largest party has particularly large bargaining power, it is more likely to form a government on its own; this effect is amplified when the party’s policy preferences are centrally located within the legislature. Empirical analysis generally indicates that in minority situations (when no singular party controls more than 50% of the seats), minimal winning coalitions are most likely to form when opposition influence is low and electoral volatility is low. These predictions will present complications to the Israeli case.

**Government Membership**

The last relevant development in the field of coalition studies relates to who gets in—who makes up a winning coalition? Much recent empirical scholarship focuses on policy preferences, which considers the policy stances of parties with regard to the preferred policies of the party that forms a coalition. One notable work in this realm by Warwick (2005) estimates the “policy horizons” of parties on multiple dimensions, or the points beyond which parties are unwilling to compromise on policies, and will, therefore, not join a coalition. In the Israeli government, policy preferences stem from identity-based differences (differences in religion, ethnic

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9 See Baron and Ferejohn (1989); Strøm and Nyblade (2009).
10 Formateur: party that is tasked with forming a coalition
11 Surplus government: One that is not minimal-winning, with more partners than necessary.
origin, and level of religious observance), which play a pivotal role in determining who joins a winning coalition and who gets which portfolios in the cabinet. To test these theories in an empirical case study, we must first understand conditions that make Israeli coalition and cabinet formation unique.

Israeli Government: An Overview

The Israeli government serves as an essential case study in coalition formation and payoff distribution. The following review will explore the institutional boundaries that affect the bargaining environment in which coalition formation occurs, explain the unique character of political parties in Israel, and explore previous scholarship on coalitions and cabinets in Israel.

The Bargaining Environment: Israeli Government Formation

In Israel, citizens vote for a certain party to be represented in the Knesset\(^\text{12}\), instead of an individual candidate. Israeli elections follow a proportional representation system of election, where each party receives a proportion of Knesset seats corresponding to its proportion of the total vote (Cox 1997). Once party representation in the Knesset is determined, a Government must be formed. Basic Law: The Government\(^\text{13}\) states:

> When a new Government has to be constituted, the President of the State shall … assign the task of forming a Government to a Knesset Member who has notified him that he is prepared to accept the task; the President shall do so within seven days of the publication of the election results, or should the need arise to form a new government; and in the case of the death of the Prime Minister, within 14 days of his death (6).

The President assigns the task of forming a majority coalition to a Knesset Member (MK) who has the best chance of forming a Government, who is usually the party leader of whichever party holds the most seats in Knesset. Since there has never been a party that holds a majority of the seats, coalition formation has been attempted after every Israeli election. The next section of Basic Law: The Government reads, “The Knesset Member to whom the President has assigned the task of forming a Government shall have a period of 28 days for the fulfilment of such task. The President of the State may extend the period by additional periods not exceeding in the aggregate 14 days (7).

Since the Knesset contains 120 seats, the MK in question must form a government coalition that holds a majority bloc of 61 seats (Mahler 2011).

Once the MK gains the support of several different parties that collectively hold 61 or more seats, he/she may present his/her government to the Knesset. A vote of confidence then occurs, where MKs demonstrate that the majority of the Knesset will support the Government. Afterwards, this new Government is in power, and the MK who formed it is appointed Prime Minister (PM). During the 28 days of negotiations, the prime minister-designate negotiates with party leaders who would consider joining a Government (Mahler 2011). Political parties will typically join a Government in exchange for the passage of party-specific legislation, control over ministries in the cabinet (portfolios), or general representation in the cabinet, which is measured by “cabinet posts.” However, conditions relating to the nature of Israeli political parties make the task of forming a coalition difficult. Particularly, the “tribalistic” nature of Israeli political parties and a strong practice of “party discipline” make government formation a difficult task.

Democratic Tribalism

As defined by Professor Acosta, democratic tribalism is “a political system based on the social and ultimately electoral competition of multiple subnational groups, which form around distinct ethnic and sectarian identi-
ties and pursue unique political identities” (268). This system of democratic tribalism “involves a range of identity groups that employ ethnicity, religiosity, and ideology to varying degrees and with disparate perspectives” (272).

Secular Ashkenazim, Mizrahim, immigrants from the former Soviet Union (FSU), Ultra-Orthodox Ashkenazim, Sephardim, and Mizrahim, national religious Jews, and Arab Israelis are notable identity groups in Israel. Referencing parties in the 35th government, it is apparent that the Former Soviet Union (FSU) constituency represented by Yisrael Beiteinu; Ultra-Orthodox Sephardim represented by Shas; Ultra Orthodox Ashkenazi represented by UTJ, and Secular Jews represented by the Blue and White party, to provide some prevalent examples (Mahler 2011).

Previous Literature Regarding Israeli Coalitions
Plenty of literature has been written about coalition behaviors in Israel, but our work will draw from three in particular that apply the aforementioned fundamental works of coalition theory (Von Neumann and Morgenstern 1944; Riker 1962; Leiserson 1968; Gamson 1961; Axelrod 1970; Browne and Franklin 1973). Though Nachmias’ “Coalition Politics in Israel” (1974) and Mahler and Trilling’s “Coalition Behavior and Cabinet Formation: The Case of Israel” (1975) are very similar works, Mahler and Trilling’s work discusses a broader set of factors that includes more recent data.

Mahler and Trilling found that, in the former years of Israeli governments, the minimal winning condition is typically evident, but not mandatory, as ideology is usually more important in determining who joins a government. Also, coalitions may include unnecessary partners to ensure the survival of the coalition, or to maintain political alliances. The minimal-connected winning condition is the most applicable of the presented theories in the Israeli case, and the number of ministries parties receive is generally a function of the number of seats they hold in the parliament, which is derived from Gamson’s Law.

Nachmias, however, concludes that empirical variations deviate from theory, as winning coalitions were mostly not minimal. Similar to Mahler and Trilling, he rationalizes these deviations by explaining that dominant parties form larger than necessary coalitions to remain dominant. Dominant parties gain the support of more parties than necessary; if they lose the support of one party, the once additional parties may now be necessary in maintaining the majority coalition. Forming non-minimal parties give these larger parties a cushion, allowing them to more easily form a majority bloc.

Going forth, we must consider that these works looked at Israel of the mid 1970s. Since then, the Likud party has become the dominant political faction, and public opinion has shifted towards the right due to an emphasis on national security popularized by the right-wing Likud. We cannot assume that coalition behaviors have remained the same, since the Likud and Labor parties form coalitions differently, and it is imperative that we revisit their conclusions to evaluate if they have held true over time.

Methodology

To test the conjectures of bargaining models of coalition theory, we must (1) Measure each party’s voting weight in the Knesset. (2) Apply the effect of the formateur’s advantage to our voting weights to calculate each party’s expected payoff (in terms of cabinet posts). (3) Examine the empirical justifications behind discrepancies.

Seat Share & Voting Weight
In order to apply theoretical specifications, we measure each party’s minimum-integer voting weight\textsuperscript{14}. Minimum-integer voting weight serves as the measure of how much bargaining power each party has in the Knesset, since minimum-integer voting weight accounts for the number of parties represented in each parliament and how many seats each party holds (and each government has a varying number of parties). Say that in a game with $n=100$ players, the quota a formateur must surpass to form a winning coalition is 51. If Party A has 30 members, their share of seats in this game always equals 30\%. Their share of voting weight differs from their seat share, as to reflect different bargaining scenarios. When three other parties control 25, 25, and 20 seats each, Party A’s share of voting weight equals 41.7\%. To calculate minimum-integer voting weight, one can enumerate all possible coalition opportunities and then search within the space for all instances in which a certain party is a critical member of the coalition, whereas a coalition would not be winning, or hold a majority without them. This task becomes difficult when many parties are involved: the Israeli governments we study have winning coalitions with 8-15 parties, so we instead utilize an algorithm that calculates the Banzhaf Index to measure the minimum-integer voting weight of each party, adopting nonlinear simplex methods first described by Straus (2003). The $r$-value of the regression of seat shares on share of voting weight is 0.963, but a few linear abnormalities do exist. For instance: in the 16th and 17th Knessot\textsuperscript{15} The Labor Party held 19 seats (about 15.8\% of the Knesset). However, since the seat distribution greatly changed, and the number of parties in parliament fell from 13 to 12, those 19 seats went from representing approximately 11.9\% of the total voting weight to 14.9\%, a roughly 3\% increase in the share of voting weight. The nonlinearities in this regression model represent the statistical and theoretical specification provided by voting weights as opposed to seat shares as a measure of how much bargaining power each party has in a government.

![Figure 1. Relationship between share of seats and voting weights in the Israeli Government, Knesset No. 13–23.](image)

Noncooperative bargaining models attempt to capture scenarios in which strategic bargaining occurs between parties; this is most definitely the case in the Israeli government—and all governments, for that matter—which require coalition formation. The Ansolabehere-Snyder-Ting (AST) model is statistically derived from the predictions of multiple different coalition formation models, including proposal-based bargaining models\textsuperscript{16}.

\textsuperscript{14} “Minimum-integer voting weight” and “voting weight” may be used interchangeably.

\textsuperscript{15} Knessot: Multiple Israeli parliaments.

\textsuperscript{16} See the Baron-Ferejohn Model (1989).
The Model

There are two underlying coalition theories that are expressed by the AST model: the competitive pricing condition and the formateur’s advantage. The competitive pricing condition suggests that the price required to gain a party’s support is proportional to their voting weight. This notion is similar to Gamson’s law of proportionality, but differs in the sense that the competitive pricing condition concerns bargaining power, as measured by the minimum-integer voting weight, instead of the empirical measure of seat shares (Ansolabehere, Snyder, and Ting 2005).

The formateur’s advantage was formalized by proposal-based models, like that of Baron-Ferejohn (1989), which link the advantage associated with being the formateur to the ability to make proposals in an environment, which most closely corresponds to closed-rule bargaining (Ansolabehere, Snyder, and Ting 2005). However, the Israeli government follows a more dynamic, open-rule of coalition bargaining, where parties in the Knesset negotiate terms of possible agreement for hours on end and differences in preference between parties—stemming from ideological divides—mean that the formateur loses some of their competitive advantage. Luckily, the AST model includes different approximations for \( c \), the price per unit voting weight, which is influenced by the type of bargaining environment. This is estimated as 1.0 when coalitions form under closed-rule bargaining and 1.2 under open-rule bargaining. This increases the cost of forming a coalition and provides a more realistic prediction about the formateur’s advantage in our open-rule environment.

The AST model discusses payoffs in terms of ‘shares-of-the-pie,’ where, according to the conditions above, the formateur is expected to receive everything minus the cost of forming a coalition (cost being represented by the percentage of the total payoff that was given to the coalition’s member-parties). Also, the formateur is expected to provide all other member-parties a share of the total payoff that is proportional to their bargaining power, as denoted by the competitive pricing condition.

Formalizing these ideas mathematically, we represent a coalition member’s share of the total payoff as equivalent to their share of the bargaining power. Therefore, we set a non-formateur’s share of the total payoff (which represents their share of the cabinet portfolios) \( x_i = cw/W \), where \( w_i/W \) represents party \( i \)’s share of the total voting weight and \( c \), which represents the price per unit voting weight, is approximated as 1.2 under the open rule of coalition bargaining. If a party \( i \) does not join the coalition, then \( x_i = 0 \), as they are not entitled to any share of the payoff.

Understanding that the cost of forming a coalition is represented by \( c \frac{W+1}{2w} \), where \( \frac{W+1}{2w} \) represents the quota needed to form a winning coalition, and \( F_i \) is a indicator of the formateur (=1 when this condition is met and =0 when it is not), we can use the following formula to predict the payoffs of all parties\(^\text{17}\), where \( Y_i \) represents a party’s predicted share of cabinet posts.

\[
Y_i = \left[ 1 - c \frac{W+1}{2W} \right] F_i + c \frac{w_i}{W}
\]

This formula will guide our calculations. In order to test the predictive power of this model, we will measure the number of cabinet posts that each coalition member should receive in the 25–34th Governments of Israel. In the 35th Government, neither the Likud nor Blue and White Party were successful in forming a winning coalition. Due to outstanding circumstances, an alternating government was agreed upon, where the Prime Ministry would rotate between the leaders of the two parties. Since the Likud and Blue and White parties were both given control over the prime ministry, they may both be considered formateurs. In this instance, \( F_i = 0.5 \), giving partial formateur status to both parties. We then compare the measured number of posts to each party’s \( Y_i \) value, which represents the share of cabinet posts predicted by nonbargaining game theories incorporated into the Ansolabehere-Snyder-Ting model.

\(^\text{17}\) In the case of the 35th government, \( F_i \approx 0.5 \) since two parties were given formateur status.
To count the number of cabinet posts each party received in a government, we reference the Knesset’s website—particularly, their page on all Israeli governments in history. Counting each party’s number of cabinet posts is difficult, since hundreds of resignations, replacements, and party changes have occurred in Israeli governments. In order to keep our count as consistent with the initial payoff distribution as possible, we set the following parameters. (1) If multiple ministers hold the same ministerial position, the first minister to be appointed is considered, and their post will belong to their respective party in our count. This gets quite complicated in Israel’s case, as one cabinet may see numerous politicians holding the same ministerial position during different periods of time. (2) Whichever party the initial minister belonged to at the time of appointment will be considered the controlling party of any cabinet post. Therefore, party changes will not affect the simulation at hand. (3) Acting Prime Ministers and other Acting Ministers will not be accounted for, since said ministers most frequently do not enter their position until after a resignation or death, and so it too is not consistent with the initial payoff distribution. This study’s statistical model cannot account for changes in the power structure after a government is formed, so we are only concerned with each government’s initial round of payoff distributions (i.e., which parties controlled how many ministries right after the government’s formation). Following Warwick and Druckman (2001) and Ansolabehere, et al. (2005), we assign a higher value to the prime ministry, which is objectively considered to be the most important ministry. All other cabinet positions are noted as a single post, but the PM is given a relative value of three posts. In the Israeli Government, the forming Government automatically gains the prime ministry after forming a majority coalition successfully. Considering these parameters, we count each party’s share of cabinet posts in each Government.

Results

Empirical Observations

After counting the number of cabinet posts each party received, we conduct the same analyses as Mahler and Nachmias. Though the dominant faction in the Israeli government has since changed (from the Labor party to the Likud party), we conclude that, according to early theories of coalition formation, coalition building in Israel has remained similar in nature. We see that the majority of coalitions in our study are not minimal-winning, but are generally minimal-connected. Regressing seat shares on the share of cabinet posts to measure payoff proportionality, we find an $r^2$ value of .716, indicating relatively strong proportionality, but nothing close to the strict proportionality proposed by Gamson’s Law. However, since it is understood that empirical applications of Gamson’s Law encounter shortfalls, we continue our study in the context of modern developments of game theory (Indridason 2015).

Application of the AST Model

When we apply the AST model to the past 10 Israeli governments, the results are quite promising. The $r^2$ value of the regression of the actual distribution of cabinet posts on the predicted distribution of cabinet posts is .877 with a correlation of .936. This indicates that there is a very strong linear relationship between the actual and predicted shares of cabinet posts, and that nearly 88% of variance is explained by the regression model.

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18 This regression is statistically significant at the .01 level
Figure 2. Relationship between the Predicted and Actual distribution of cabinet posts in the 25th–35th Israeli government.

However, our model falls short in accurately predicting the formateur’s payoff. It is important to note that the logic underlying the formateur’s advantage stems from the leverage gained in a closed-game, where the formateur has the unique ability to make proposals for coalition membership that other parties can either agree to or reject (Baron 1989), and the Israeli government follows open-rule bargaining. While the AST compensates for this by increasing the approximated price per voting weight, the residuals of the formateurs’ predictions are far greater than those of all other parties, and the correlation between the actual and predicted share of cabinet posts is .737, which is much lower than that of all parties. The high residuals, which can be visualized by the points in the upper-right corner of our graph, represent the reality that our predictions for the formateur’s payoff were much further from the truth than those of other member-parties. This supports our hypothesis that the formateur’s advantage, while present, is overestimated by bargaining models, as the extent of the advantage is heavily determined by the context in which governments are formed. Understanding this, we will now examine literature on each election that corresponds to an instance of government formation, cross-analyzing how situational factors influence the extent of the formateur’s advantage and other deviations from our predictions.

Empirical Deviations and the ‘Why?’

Israel provides a unique environment in which to explore cases in which theoretical limitations and confounding variables manifest themselves, and an analysis of Israel can contribute to a better understanding of parliamentary democracies whose instances of coalition formation deviate from theory. Since the formation of the 25th Israeli Government, much has happened in Israel, including but not limited to: the assassination of PM Yitzhak Rabin, the First and Second Intifadas, the first democratic election in a newly-sovereign Gaza, and the direct election of Prime Minister in the 1996 and 1999 elections (which correspond to the 27th and 28th Governments). To understand how external factors, like person-politics and stark identity differences impact Government formation and the distribution of cabinet posts, we reference literature from the Israeli Democracy Institute on each election and its corresponding instance of Government formation. We then cross-analyze these events with our findings, comparing abnormal political behaviors to discrepancies in our predictions.
<table>
<thead>
<tr>
<th>Gov. No.</th>
<th>IDI-Sourced Contextualization</th>
<th>Manifestation in the Government</th>
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<tr>
<td>25, 26</td>
<td>The election results left the Labor Party with a preventive bloc of 61, together with Meretz and the Arab parties, but Rabin rejected forming a government with the Arab parties and managed to convince Shas to join his coalition.</td>
<td>Rabin’s refusal to coalesce with Arab parties forced him across the aisle, to the orthodox Shas party. This identity-fueled move artificially increased the cost of Shas’ membership, giving them a competitive advantage that led to a greater than anticipated share of posts.</td>
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<td>27</td>
<td>The results of the Knesset election made it clear that the two ballot system was harmful for the large parties. The power of the sectoral parties increased. A new sectoral party, representing the recent wave of immigrants from the former Soviet Union—Yisrael B’Aliya, headed by Natan Sharansky—won seven seats. The Arab parties were greatly strengthened and received nine seats together, and the religious parties were also strengthened, and together received 23 seats in the new Knesset.</td>
<td>In the elections that correspond to the 27th, 28th, and 29th governments, the Prime Minister was directly elected. This periodic, institutional change decreased the power of the formateur, causing a substantial decrease in the formateur’s advantage. However, the extent of this change is further exemplified in the next government, where the directly elected Prime Minister represented a party with less influence as the Likud party, which has been a predominant power in the Knesset since the 1970s.</td>
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<td>28, 29</td>
<td>These elections, which were the second elections to be held under the rules of the direct election system, mark the peak of parliamentary fragmentation, the collapse of the large parties and the strengthening of sectoral parties. With his parliamentary group holding less than a quarter of the Knesset seats, it was clear that the coalition that Ehud Barak would form would include a large number of parliamentary groups, and that its standing as a ruling party would be weak.</td>
<td>Ehud Barak’s One Israel party held approximately 22% of the seats in Knesset, which translated to about a quarter of the total voting weight. Our model predicted a formatuer’s advantage that allocated 69.7% of the cabinetry to One Israel, though the strong fragmentation between parties, alongside the lack of traditional bargaining power held by the formateur, led to a nonexistent advantage for the forming party, which held about 27% of the cabinetry.</td>
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<td>30</td>
<td>After more than two years of the Intifada, many people felt that the Oslo process had exhausted itself, and that there was no partner for peace. The Labor Party declined to 19 seats, while Meretz received only six seats. Many of those disappointed with the left moved to the center of the political map, and voted for Shinui.</td>
<td>Since public opinion had shifted to the right-wing, the Likud had garnered nearly one third of the seats in the Knesset. Since the Likud formed the 30th government with much influence, they took about 55% of the cabinetry for themselves. While this indicates an advantage, it is not near the 91% that is hypothesized by our model. The Likud likely gave more posts than necessary to their ally parties and miscellaneous MKs in order to maintain strong ties, which would aid them in future instances of coalition formation.</td>
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</table>
The previous Knesset served under the shadow of the withdrawal from the Gaza Strip in 2005. Prime Minister Ariel Sharon (Likud) got the plan approved by the Knesset, but with great difficulty, in view of the strong opposition within his own party. He announced his breaking away from the Likud (of which he had been one of the founders) and the establishment of a new centrist political framework. The elections were set for March, 2006, and the early public opinion surveys predicted a landslide victory for Sharon and Kadima.

The disruption in Sharon’s interpersonal relationships within the Likud party, as a result of policy disagreement, sparked the creation of Kadima. A formateur’s advantage is present here, but not to the extent hypothesized by our model. We hypothesize that due to his abandonment of the Likud party, Sharon’s cost of gaining the support of Likud had been artificially increased, which explains the discrepancy in how many posts Likud received. They were predicted to receive around 17% of the cabinetry, but actually received 27%. This is also present in the Shas party, a long-time ally of the Likud, who received 18% of the cabinetry, as opposed to the hypothesized 11%.

True to his word, Olmert submitted his resignation to the President, who called upon Livni to form a new government. The coalition negotiations that Livni held failed, and after the President realized that there was no majority for an alternative government, early elections were called for February 2009. The Zionist left-wing parties continued to decline. Labor shrank to 13 seats (the fourth largest party), and Meretz received only three seats. Yisrael Beytenu did very well with 15 seats, Kadima won 28 seats over Likud’s 27. Livni demanded the opportunity to form a government, but it was clear that Likud would have a better chance of doing so. Aside from a 12% discrepancy in the formateur’s advantage, our model very accurately predicted how much of the cabinet each party would be entitled to. The correlation between the predicted and actual share of cabinet posts each party received is .9982.

The process of government formation seems to have been particularly difficult this time, primarily because of the alliance between Yair Lapid and Naftali Bennett—a close cooperation established between the leaders of the two senior coalition partners Yesh Atid and The Jewish Home. The weight of these two parties combined—31 seats—is equal to that of Likud-Beiteinu. When the prime minister's party is a minority in the government, the constraints of the coalition increase substantially, since the coalition partners have a strong bargaining position.

In this Government, the Likud party saw a notable formateur’s advantage, receiving 69% of the cabinetry. In this case, The Jewish Home saw a slight increase in their distribution of cabinet posts, receiving slightly more of the cabinetry than anticipated. While the formateur had less bargaining power in this Government than anticipated, no glaring discrepancies from our model were found.

The main reason for Prime Minister Netanyahu's decision to dissolve the government and call early elections was, apparently, a deep personal crisis of confidence between him and two of his senior ministers: Finance Minister Yair Lapid (Yesh Atid) and Justice Minister Tzipi Livni. An interpersonal trouble between Netanyahu and the leaders of Yesh Atid and Hatnua led him to dissolving the Government. This is unconventional and goes to show how the relationship between party leaders can influence the stability of a Government. While negotiating the formation of
Hatnua. Netanyahu fired these two ministers in early December, 2014. Negotiations with other parties took a slow pace and in a surprising move, Avigdor Liberman, leader of Yisrael Beytenu, decided not to join the coalition due to his personal lack of confidence in Netanyahu. This move reshuffled the dynamics of the coalition-building process, and gave Shas and the Jewish Home a considerably strong bargaining position vis-à-vis the Likud.

The incoming Netanyahu-Gantz government, each taking their turn as prime minister, is unique in many respects. It is based on the principle of inter-bloc parity and on a mechanism for rotation in the position of prime minister—a mechanism that has created a new constitutional role of “alternate prime minister.” The bloc of parties opposing the formation of a government with Netanyahu at its helm, gained 62 seats in total, but the large ideological gaps among its component parties - prevented them from coming together to form a government.

Unforeseen circumstances brought about by the novel COVID-19 pandemic led to the creation of the 35th government. The Blue and White party and the Likud party agreed to an alternating prime ministry, where Netanyahu would begin as Prime Minister, with Gantz taking over the position at a later date (this alternation did not occur; since the Knesset failed to agree on a budget, this Government was dissolved). Since both Likud and Blue and White had partial control over the prime ministry, we made the indicator of the formateur in our model = 0.5, assigning that value to the Likud and Blue and White parties. As is consistent with the rest of our data, the model overestimated the formateur’s advantage, but assigning the split indicator value created a more realistic image. Where the Likud’s payoff was overestimated by 10% and Blue and White’s considerably more, though their share of voting weight was initially lower.

Figure 3. IDI-sourced contextual reasoning behind predictive discrepancies

One quintessential example allows us to understand the implications of what goes on behind the scenes, interpersonally or identity-based—in this case, identity-based. We reference the election that corresponds with the 25th and 26th governments. The Labor party could have formed a winning coalition with Meretz and Arab parties, but identity-based differences prevented Rabin from coalescing with Arabs, so he turned to Shas for coalition membership. Rabin’s reliance on Shas artificially increased their bargaining power, leading to Shas gaining a disproportionately large share of cabinet posts\(^\text{19}\). The rest of our analysis confirmed that discrepancies in our predictions can nearly always be explained by empirical mechanisms at play behind the scenes, and that the situational context in which a government forms can typically provide insight into what the general cost of forming a coalition will be. These real-world rationalizations provide valuable evidence regarding why it is important to study coalition theory in the context of empirical instances of coali-

\(^{19}\) Disproportionate to what was predicted by our model.
tion formation: they explain to us why theories work and why they don’t, often giving important insights into what could be done to improve the theoretical framework.

**Limitations and Conclusion**

While political coalitions have served as a consistently reliable medium through which to study coalition theory, one notable limitation has not been uniformly addressed: the value of cabinet portfolios. Our research referred to spots in the cabinet as “posts” referring generally to the people who are represented in the cabinet, since this terminology focuses less on each constituency. We follow a weighted voting system that gives added value to the Prime Ministry, but the rest of the portfolios share the same value of 1. However, different cabinet portfolios have different subjective values to each party. For example, the Shas party holds the Ministry of Religious Affairs to a higher value than the Ministry of the Interior, while the Labor party is more concerned with the Ministry of the Interior than the Ministry of Religious Affairs. It may be difficult to measure the value of each cabinet post to each party, but doing so empirically could establish a framework for theoretical progress to be made in this hemisphere.

Our study focused on 10 instances of coalition building in the Israeli government. This allowed us to deeply assess each coalition process and their corresponding Government. While the Israeli government is an idiosyncratic system, it is not the only government that can serve as a unique case study, for every government is idiosyncratic in its own way. Looking at coalitions in an understudied, developing parliamentary system, such as budding Balkan and other Eastern European democracies, can allow us to evaluate these countries’ oddities and use them to understand further empirical implications of the theoretical framework.

We used the Ansolabehere-Snyder-Ting model to assess how bargaining games can predict the real-world outcome of government formation. It’s purpose is to predict the share of cabinet posts given to a party, given their voting weight, and to do so under the assumption that the formateur is entitled to a significant advantage due solely to the fact that they formed the government. However, this is just one game. Implementing numerous game-scenarios into parliamentary coalition formation could allow us to better understand which games are most applicable to certain types of governmental systems, and would offer a closer look at which games can be used to overcome country-specific idiosyncrasies.

Nevertheless, our findings are promising. Disregarding the extent of the formateur’s advantage, our model predicted to a high degree of certainty the distribution of cabinet posts in Israeli governments 25-35. We apply game theory to political institutions to better understand why and how governments form, and understanding the mechanisms behind cabinet post distributions further enhances our ability to predict what will happen in future governments. Our research explores the empirical reasoning behind instances in which game theory failed to accurately predict what truly happened. We can gather from our qualitative cross-analysis that understanding the situational context, especially in regards to identity-based differences and interpersonal relations between party leaders, allows us to understand why the real-world deviated from theory, and further integration of empirical evidence into the development of the theoretical framework will allow for more accurate theories that yield more accurate predictions.

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