$$p(\nu_1 \rightarrow \nu_2) = sin^2(2\theta)sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right) = sin^2(2\theta)sin^2\left(1.27\frac{\Delta m^2[eV]L[km]}{E_\nu[GeV]}\right) \qquad (1)$$

Eq. (1) states the function for calculating the probability of neutrinos converting their flavor from their beginning status to another status in a two-neutrino problem based on distance. The $sin^2(2\theta)$ in the equation defines the amplitude, or the maximum possibility of a conversion happening, while the Δm^2 is the difference of the square of the mass of the particles. L states the distance the neutrino particles travel in kilometers and is the main variable of this function.

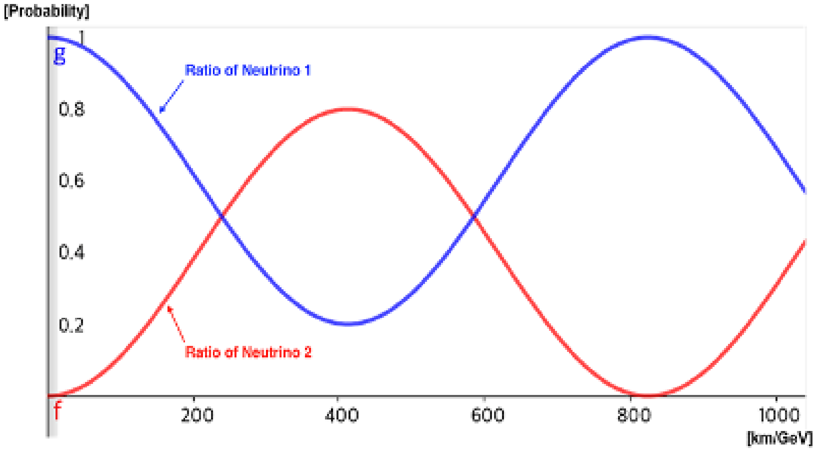


Figure 1. Neutrino oscillation between two different neutrinos. Probability of neutrino 1 is decreasing while probability of neutrino 2 is increasing when neutrino is traveling in a certain distance

A graphical rendition of the function with the values shown below substituting the various parameters; $m^2 = 0.003$ $eV^2, sin^2(2\theta) = 0.8, E_\nu = 1GeV$ in Eq. (1) is shown in Fig. 2, where the possibility of neutrino changing flavor is shown to oscillate in distance.

Relationship between Neutrino Oscillations and Baryon Asymmetry

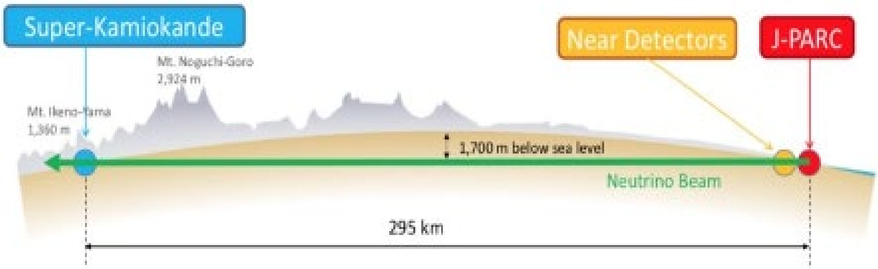
As explained in Sakharov's Proposed Conditions for Baryon Asymmetry, it is hypothesized that three conditions need to be met in order for baryon asymmetry to occur; violation of baryon number, violation of C-symmetry and CP-symmetry, and the violation of thermal equilibrium.

In order to explain the discrepancy between the Standard Model and the observed unbalance between matter and antimatter particles in the current universe, scientists have been conducting experiments to uncover whether violation of CP-symmetry was observable in the formation and interactions of matter and antimatter particles. So far, the CP-symmetry violation observed in quarks has been insufficient to explain the difference in the ratio of naturally observed matter and antimatter particles. However, a discrepancy has been found in the neutrino/antineutrino oscillations, where scientists suspect that the rate of oscillations differ and thus creating a difference in the rate matter and antimatter are formed from interactions between basic particles, thus opening a possibility that the asymmetry in the rate of oscillations in neutrino and antineutrino could be the cause of the baryon asymmetry observed in the universe today. This led the scientists to conduct experiments with neutrino and antineutrino beams to investigate whether the rate of oscillations differ in the two types of the same particle.

Various Neutrino Experiments

The Tokai to Kamioka (T2K) Experiment

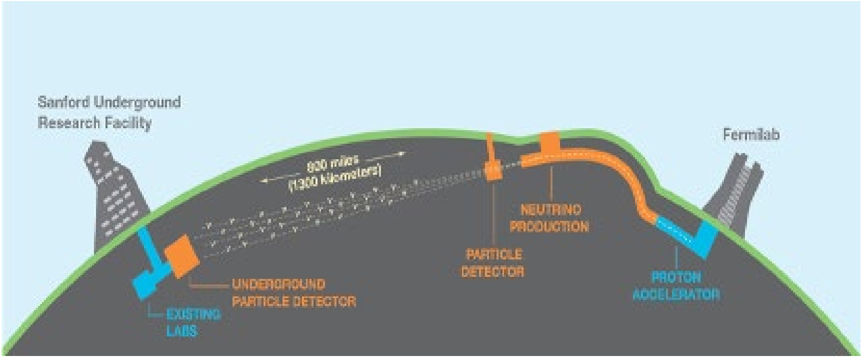
The Tokai to Kamioka Experiment, more commonly called the T2K Experiment, is an experiment for investigating the oscillations of neutrinos. Built 1km underground in order to minimize outer neutrino interference, the Super Kamiokande detector contains 50000 tons of water, and has 13000 photomultiplier tubes installed. As a neutrino particle interacts with the water inside the detector, it causes Cherenkov radiation, a phenomenon where particles move faster inside a certain medium than the speed of light inside that medium. The photomultiplier tubes detect light from the Cherenkov radiation and analyze the patterns shown in the light emission. A muon neutrino interacts with particles to create muons, which in turn cause a ring-shaped Cherenkov light emission with well-defined edges. On the other hand, an electron neutrino creates relatively lighter electrons, which are more prone to scattering and thus create ring-shaped light emission with fuzzy edges. Using this principle, the T2K experiment uses either a muon neutrino beam or a muon antineutrino beam created in Tokai, which is then directed towards the Super Kamiokande detector 295km away. The T2K experiment is designed to detect neutrinos at the low energy range of around 600 MeV.



As mentioned earlier in the paper, a team of researchers at T2K announced in April that through T2K they had managed to exclude half of the possible values in the difference between neutrino oscillations and antineutrino oscillations, with statistics pointing to a high possibility that more electron neutrinos emerged than their antimatter counterparts as a result of neutrino/antineutrino oscillations. In this experiment, the team used the accelerator to launch 1.49×10^{21} and 1.64×10^{21} protons in neutrino beam mode and antineutrino beam mode, respectively. They then analyzed the observed neutrino detection events and determined the number of detected electron neutrinos and electron antineutrinos. Prior to the experiment, the researchers had predicted that they would detect 82 electron neutrinos and 17 electron antineutrinos for maximal neutrino enhancement $(\delta_{cp} = -90^{\circ})$ and 56 electron neutrinos and 22 electron antineutrinos for maximal antineutrino enhancement $(\delta_{cp} = 90^{\circ})$. In the experiment, the researchers detected 90 electron neutrinos and 15 electron antineutrinos, proof that the conditions of the neutrino oscillations were closer to the conditions of maximal neutrino enhancement. The research team concluded that this result provided enough statistical evidence to exclude nearly half of the possible values for δ_{cp} with a confidence level of 3σ, with the exclusion eliminating most of the possibilities of an antineutrino enhancement. From this, the research team concluded that it was likely that CP-symmetry was violated in the neutrino oscillation, yet that conclusion could not be ascertained with a high enough confidence level. The research team at T2K planned to heighten the confidence level by upgrading the equipment and detectors for measurements in the near future.

DUNE/LBNF

The DUNE/LBNF is an experiment that is scheduled to be conducted in the United States, where a beam of neutrino particles will be launched from Fermilab to Deep Underground Neutrino Experiment at Sanford, South Dakota, 1300 km away from the neutrino beam source. The experiment is expected to bring a deeper understanding of the nature of the neutrino oscillation, as well as properties of neutrinos that are yet to be uncovered. Such properties include, but are not limited to, the existence of a sterile neutrino, the mass of the three flavors of neutrinos, and so on. The DUNE/LBNF experiment will be using the most intense neutrino beam to be used in a neutrino experiment which is expected to increase the precision of the experiments conducted. It is also expected to be able to adjust the energy level of the neutrino beams, which will help in investigating how neutrinos behave differently based on energy levels. The detectors will be placed 1.3km underground, with 70000 tons of liquid argon filled into the detector tanks. The cold liquid argon is expected to enhance the detector's capabilities to capture the particle interactions occurring inside the detector in greater detail.



Discussion

General Comparison of T2K and DUNE/LBNF

In the previous chapter, characteristics and goals of two experiments, the T2K experiment and the planned DUNE/LBNF experiment, has been discussed. Both experiments aim to uncover the answers to the question of baryon asymmetry through finding CP-symmetry violation in neutrino oscillations. However, there are several differences in the two experiments. For one, the distance between the neutrino source and the far detector are very different; the distance from Tokai to Kamiokande is 295 km, while the distance from Fermilab in Chicago to DUNE in South Dakota is 1300 km. Another difference is the energy level used for the neutrino beams. T2K uses a relatively low energy neutrino beam of 600 MeV, while DUNE/LBNF is planning to use a higher energy neutrino beam of 1~5 GeV. One more major difference would be the detection medium used in the detectors of the two experiments. T2K uses 50000 tons of purified water, while DUNE/LBNF is planning to use 70000 tons of liquid argon. An organized comparison of these differences in the experimental factors is shown in Table 1.

Table 1. Comparison of particle accelerator experiments in T2K and DUNE/LBNF

	T2K	DUNE/LBNF
Distance between Neutrino Source and Detector	295 km	1300 km

Energy Level of Neutrino Beam	600 MeV	1~5 GeV
Detection Medium	50000 tons of Purified Water	70000 tons of Liquid Argon
Value of Δm^2 for Optimal Observations	2.51×10^{-3} eV^2	2.85×10^{-3} eV^2

The Locations of the Neutrino Detectors

One of the experimental factors of interest is the two experiments' distance between their respective neutrino source and the far detector. As mentioned earlier, the distance from Fermilab to DUNE is more than 4 times the distance between Tokai and Kamiokande. It is thought that the reason for this is the difference in the energy level of the neutrino beams. As shown in Eq. (1), the oscillation of neutrinos based on distance depends on the energy level of the neutrino beam. The higher the energy level, the longer the distance traveled per oscillation probability cycle. Based on this, it can be concluded that the neutrinos from the DUNE/LBNF experiment would have to travel a longer distance in order to finish one probability cycle. As shown in Fig. 3 and 4, detector locations for T2K and DUNE experiments were set at the positions where neutrino changing ratio is the highest. This is because those positions are the best position for detecting neutrino oscillations.

Calculation and Speculation of $\boldsymbol{\Delta m^2}$

As shown in Table 1, the distance used in T2K is 295 km, while the distance used in DUNE/LBNF is 1300 km. As mentioned, assuming that the location of the detectors for each experiment is in the optimal location for detection of electron neutrinos, the value of Δm^2 can be calculated by substituting the variables in Eq. (1) with values that are already known.

Differentiating Equation 1 produces Eq. (2):

$$\frac{dp}{dL} = sin^2(2\theta) \times 2\sin\left(1.27\frac{\Delta m^2 \cdot L}{E_\nu}\right) \times \cos\left(1.27\frac{\Delta m^2 \cdot L}{E_\nu}\right) \times 1.27\frac{\Delta m^2}{E_\nu} = 0 \qquad (2)$$

Then, Eq. (2) becomes

$$cos\,(1.27\frac{\Delta m^2 \cdot L}{E_\nu}) = 0 \qquad (3)$$

For the T2K experiment, assuming that the value of $sin^2(2\theta)$ is set to 0.8 like it was done in Fig. 1 and inserting 600MeV for the E_ν in the denominator of Eq. (3) produces a function based on Δm^2 and L, with the local extrema having to be at $L = 295km$. Calculating for Δm^2 yields 2.52×10^{-3} eV^2as its value. Doing the same calculation for DUNE/LBNF yields the value 2.85×10^{-3} eV^2 as the value of Δm^2.

As seen from the calculations, there is a discrepancy in the values of Δm^2derived from the assumption that the two detectors are placed in locations where the probability of observing neutrino oscillations is the highest.

Based on these calculations, a graphical rendition of the probability function of neutrino oscillations based on distance has been shown as Fig. 5 and Fig. 6.

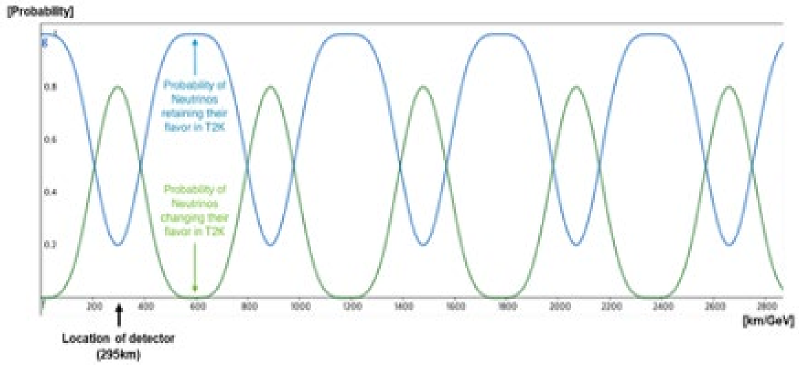


Figure 5. Probability of neutrino oscillations based on travel distance in T2K experiment. Maximum oscillation between neutrino 1 and neutrino 2 is happening at 295km

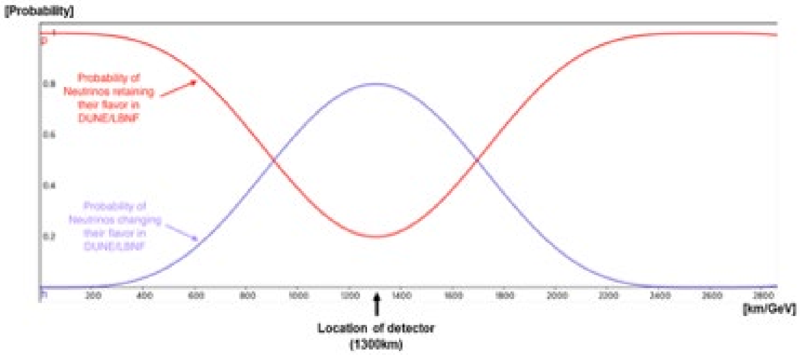


Figure 6. Probability of neutrino oscillations based on travel distance in DUNE/LBNF experiment. Maximum oscillation between neutrino 1 and neutrino 2 is happening at 1300km

The discrepancy of the calculated values of Δm^2 could be due to a number of factors. For example, the exact value of Δm^2 is unclear as of present, although it has been confirmed that it is somewhere within the range of 10^{-3} eV^2. Also, the principal assumption behind the calculations was that the detectors were on the exact locations that guaranteed the optimal conditions for observing neutrino oscillations—this may not be true in reality, as there are other things to be considered when constructing a massive neutrino detection chamber.

As shown in Fig. 7, it has been experimentally shown that there are three mass states of neutrinos. These mass states each have a different probability of becoming unique flavors, with different energy levels for each mass state. These mass states are each referred to as ν_1, ν_2, ν_3 with ν_1 having a ⅔ chance of interacting to become an electron neutrino, and a ⅓ chance of becoming a muon or a tau neutrino. ν_2 has roughly equal probability of becoming either of the three flavors. ν_3 has a 45% probability of becoming a muon neutrino and a 45% probability of becoming a tau neutrino, and a 10% chance of becoming an electron neutrino. The difference between the energy levels for each mass state has been measured from various observations of interactions of neutrinos occurring in the Sun and Earth's atmosphere. Through this, the difference between ν_1 and ν_2, as well as the difference between ν_2 and ν_3 have been measured, from which it was concluded that the Δm^2 value for ν_1 and ν_2 was on the scale of 1×10^{-5} eV^2, while the Δm^2 value for ν_2 and ν_3 was estimated to be 400 times greater than the Δm^2 value for ν_1 and ν_2. From this, it can be deduced that the value of Δm^2 calculated earlier in this paper is the value of Δm^2 for the difference between either ν_2 and ν_3, or ν_1 and ν_3. This, combined with the information shown in Fig. 7, leads to the conclusion that it is likely that ν_3 is very closely related to the mass of the muon neutrino, as the muon neutrino is the original state of neutrinos in the accelerator experiments mentioned earlier.

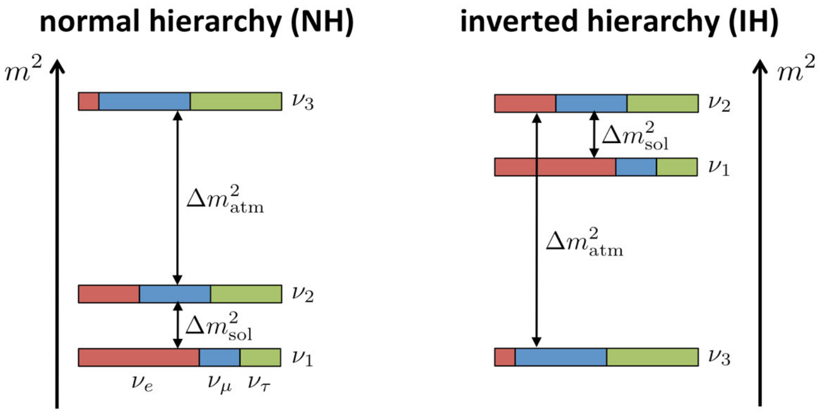


Figure 7. Hierarchy of Neutrino mass states and Δm^2 between different neutrinos. Each of three different neutrino phases (Muon, tau and electron neutrinos) has three different mass states[10]

Conclusion

This research began in order to understand how neutrino oscillations could be the answer to explaining baryon asymmetry. Through understanding other researches about neutrino oscillations and CP-symmetry violation, it was understood that by proving neutrino oscillations violate CP-symmetry, the problem of baryon asymmetry in the universe could be answered.

Then two major neutrino experiments, T2K and the currently-planned DUNE/LBNF, were compared to each other. Through this comparison, it was concluded that these two experiments placed their detectors in locations where the neutrino oscillation could be relatively easily detected with the highest oscillation probability. Based on this information, calculating the approximate Δm^2 was attempted, with the values derived from the distance between the neutrino beam source and the detector and the energy level of the beam showing the Δm^2 to be near the order of 10^{-3} eV^2, although the two values were different from each other. This confirms that the neutrino oscillation detected in these experiments is from muon neutrinos to electron neutrinos which is designed for the experiments.

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